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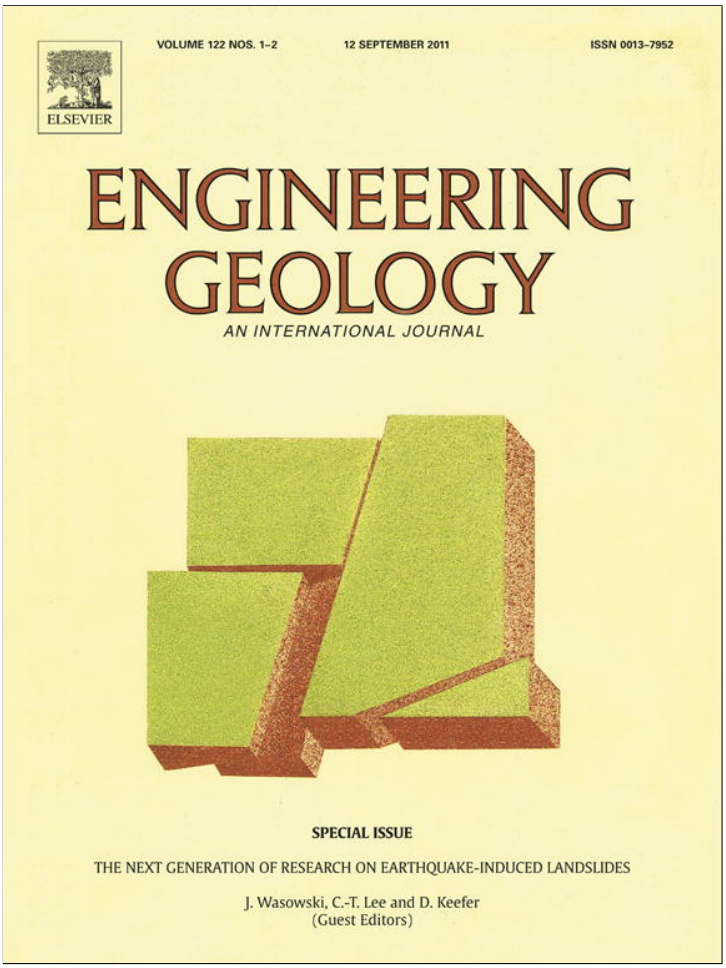

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Engineering Geology 122 (2011) 43–50

Contents lists available at ScienceDirect

Engineering Geology

journal homepage: www.elsevier.com/locate/enggeo

Methods for assessing the stability of slopes during earthquakes—A retrospective

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ARTICLE INFO

Article history:
Received 10 February 2010
Received in revised form 23 June 2010
Accepted 26 September 2010
Available online 1 October 2010

Keywords:
Earthquake
Landslide
Seismic hazards
Seismic slope stability

ABSTRACT

During the twentieth century, several methods to assess the stability of slopes during earthquakes were developed. Pseudostatic analysis was the earliest method; it involved simply adding a permanent body force representing the earthquake shaking to a static limit-equilibrium analysis. Stress-deformation analysis, a later development, involved much more complex modeling of slopes using a mesh in which the internal stresses and strains within elements are computed based on the applied external loads, including gravity and seismic loads. Stress-deformation analysis provided the most realistic model of slope behavior, but it is very complex and requires a high density of high-quality soil-property data as well as an accurate model of soil behavior. In 1965, Newmark developed a method that effectively bridges the gap between these two types of analysis. His sliding-block model is easy to apply and provides a useful index of co-seismic slope performance. Subsequent modifications to sliding-block analysis have made it applicable to a wider range of landslide types. Sliding-block analysis provides perhaps the greatest utility of all the types of analysis. It is far easier to apply than stress-deformation analysis, and it yields much more useful information than does pseudostatic analysis.

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1. Introduction

Methods for assessing the stability of slopes during earthquakes have evolved steadily since the early twentieth century, when the first attempts at modeling the effects of seismic shaking on slopes were developed. These early efforts, based simply on adding an earthquake force to a static limit-equilibrium analysis, were formalized by Terzhagi (1950) and comprise what came to be known as pseudostatic analysis. Soon after, finite-element modeling, a type of stress-deformation analysis, was developed and eventually would be applied to slopes (Clough and Chopra, 1966). In these early years, however, this type of analysis was profoundly complex and computationally daunting. Newmark (1965) proposed a method for estimating the displacement of slopes during earthquakes that addressed some of the crude assumptions of pseudostatic analysis but was still quite simple to apply in practice and thus overcame the difficulties of early stress-deformation analysis. Subsequent researchers refined Newmark's analysis to allow for more complex and realistic field behaviors, which gave rise to so-called decoupled and fully coupled displacement analyses. Interestingly, however, these types of analyses—from pseudostatic to fully coupled displacement—did not successively replace one another but rather co-exist in the practicing community. This creates a level of confusion and uncertainty because different analyses in many instances yield significantly different results. Therefore, both practitioners and researchers must understand the technical underpinnings and limitations of these various types of analysis so that they can apply the appropriate type of analysis to the conditions being studied.

Methods developed to date to assess the stability or performance of slopes during earthquakes thus fall into three general categories: (1) pseudostatic analysis, (2) stress-deformation analysis, and (3) permanent-displacement analysis. Each of these types of analysis has strengths and weaknesses, and each can be appropriately applied in different situations. The sections that follow describe the historical development of each of these families of analysis and discuss their advantages and limitations. The paper concludes with a discussion of appropriate applications of these types of analysis.

2. Pseudostatic analysis

Stability analyses of earth slopes during earthquake shaking were initiated in the early twentieth century using what has come to be known as the pseudostatic method; the first known documentation of pseudostatic analysis in the technical literature was by Terzhagi (1950). Pseudostatic analysis models the seismic shaking as a permanent body force that is added to the force-body diagram of a conventional static limit-equilibrium analysis; normally, only the horizontal component of earthquake shaking is modeled because the effects of vertical forces tend to average out to near zero. Consider, for example, a planar slip surface in a slope consisting of dry, cohesionless material (Figure 1). The pseudostatic factor-of-safety equation is

$$FS = [(W\cos\alpha - kW\sin\alpha)\tan\phi] / (W\sin\alpha + kW\cos\alpha) \quad (1)$$

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0013-7952/$ – see front matter. Published by Elsevier B.V.
doi:10.1016/j.enggeo.2010.09.017

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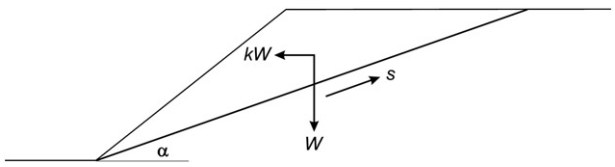

Fig. 1. Force diagram of a landslide in dry, cohesionless soil that has a planar slip surface. W is the weight per unit length of the landslide, k is the pseudostatic coefficient, s is the shear resistance along the slip surface, and α is the angle of inclination of the slip surface.

where FS is the pseudostatic factor of safety, W is the weight per unit length of slope, α is the slope angle, ϕ is the friction angle of the slope material, and k is the pseudostatic coefficient, defined as

$$k = a_h / g \tag{2}$$

where a_h is the horizontal ground acceleration and g is the acceleration of Earth's gravity.

A common approach to using pseudostatic analysis is simply to iteratively conduct a limit-equilibrium analysis using different values of k until $FS = 1$. The resulting pseudostatic coefficient is called the yield coefficient, k_y. In the simplest sense, any ground acceleration that exceeds $k_y \times g$ is defined as causing failure. The obvious simplicity of pseudostatic analysis as an extension of static limit-equilibrium analysis makes it easy to apply, and it came into broad usage rather quickly.

Pseudostatic analysis has some obvious drawbacks; Terzhagi acknowledged that "the conception it conveys of earthquakes on slopes is very inaccurate" (1950, p. 90). Foremost among these drawbacks is that including the earthquake shaking as a permanent, unidirectional body force is extremely conservative: it assumes that the earthquake force is constant and acts only in a direction that promotes slope instability. For this reason, pseudostatic coefficients generally are selected to be some fraction of the peak acceleration to account for the fact that the peak acceleration acts only briefly and does not represent a longer, more sustained acceleration of the landslide mass.

Selection of the pseudostatic coefficient is thus the most important aspect of pseudostatic analysis, but it is also the most difficult. Table 1 lists several recommendations for selecting a pseudostatic coefficient; significant differences in approaches and resulting values clearly exist among the studies cited. One key issue is calibration: some of these studies were calibrated for earth-dam design, in which as much as 1 m of displacement is acceptable. And yet these same values commonly are used in the stability assessment of natural slopes, in which the acceptable displacement might be as little as 5–30 cm (Wieczorek et al., 1985; Blake et al., 2002; Jibson and Michael, 2009). The most commonly used values are probably similar to the $k = 0.15$ and $FS > 1.1$ in general use in California (California Division of Mines and Geology, 1997), but again, these criteria were formulated for earth dams that could accommodate about 1 m of displacement.

After summarizing a number of published approaches for determining an appropriate seismic coefficient, Kramer (1996, p. 436–437) concluded that "there are no hard and fast rules for selection of a pseudostatic coefficient for design." Justifications cited by practitioners in the engineering community of why a certain pseudostatic coefficient is used generally are some variation of, "We have always done it that way," or "It seems to work." Suffice it to say that in the practicing community at large a rigorously rational basis for selecting a pseudostatic coefficient remains elusive, and engineering judgment along with standard of practice generally are invoked in the selection process.

Two recent studies have attempted to rationalize the selection of the pseudostatic coefficient. Stewart et al. (2003) developed a site screening procedure, based on the statistical relationship of Bray and Rathje (1998), wherein a pseudostatic coefficient is calculated as a function of maximum horizontal ground acceleration, earthquake magnitude, source distance, and two possible levels of allowable displacement (5 and 15 cm). Bray and Travasarou (2009) presented a straightforward approach that calculates the pseudostatic coefficient as a function of allowable displacement, earthquake magnitude, and spectral acceleration. The common basis of these rationalized approaches is calibration based on allowable displacement.

As stated previously, pseudostatic analysis tends to be over-conservative in many situations, but there are some conditions in which it is unconservative. Slopes composed of materials that build up significant dynamic pore pressures during earthquake shaking or that lose more than about 15% of their peak shear strength during shaking are not good candidates for pseudostatic analysis (Kramer, 1996). In fact, several case studies exist of dams that passed a pseudostatic stability analysis but failed during earthquakes (Seed, 1979).

One last limitation of pseudostatic analysis is that, because it is a limit-equilibrium analysis, it tells the user nothing about what happens after equilibrium is exceeded. The analysis shows a slope to be either stable or unstable, but the consequences of instability, or even the likelihood of failure, cannot be judged.

In summary, pseudostatic stability analysis is easy to use, has a long history that provides a body of engineering judgment regarding its application, and provides a simple, scalar index of stability. But this simplicity stems from a rather crude characterization of the physical processes of dynamic slope behavior that produces several drawbacks, including difficulty in rationally selecting a pseudostatic coefficient and in actually assessing the likelihood or results of failure. The use of pseudostatic analysis, while still widespread, gradually is being supplanted by the more sophisticated types of analysis discussed in the following sections.

3. Stress-deformation analysis

In 1960, Ray Clough at the University of California, Berkeley, developed and named the finite-element method of engineering analysis (Clough, 1960). This method was based on mathematical methods first developed by Richard Courant (1943). Finite-element modeling uses a mesh to model a deformable system; the deformation at each node of the mesh is calculated in response to an applied stress (Figure 2). This method soon began to be applied to slopes, particularly earth dams, and it provided a valuable tool for modeling the static and dynamic deformation of soil systems. Subsequent developments in stress-deformation analysis have yielded a family of analytical methods, including finite-difference, distinct-element, and discrete-element modeling.

Table 1
Pseudostatic coefficients from various studies.

Investigator	Recommended pseudostatic coefficient (k)	Recommended factor of safety (FS)	Calibration conditions
Terzhagi (1950)	0.1 (R-F = IX) 0.2 (R-F = X) 0.5 (R-F > X)	>1.0	Unspecified
Seed (1979)	0.10 (M = 6.50) 0.15 (M = 8.25)	>1.15	<1 m displacement in earth dams
Marcuson (1981)	0.33–0.50 × PGA/g	>1.0	Unspecified
Hynes-Griffin and Franklin (1984)	0.50 × PGA/g	>1.0	<1 m displacement in earth dams
California Division of Mines and Geology (1997)	0.15	>1.1	Unspecified; probably based on <1 m displacement in dams

R-F is Rossi-Forel earthquake intensity scale.
M is earthquake magnitude.
PGA is peak ground acceleration.
g is acceleration of gravity.

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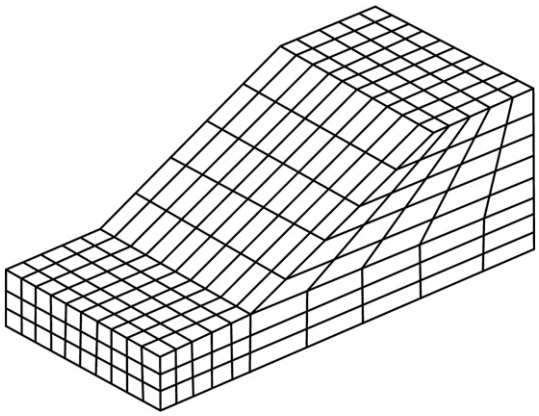

Fig. 2. Example of a 3-D mesh of a slope that could be used in a stress-deformation analysis.

Several applications of finite-element modeling to earth structures have been developed and published; Kramer (1996) provided a good summary of various methods and their associated studies. Seed et al. (1973) analyzed the failures of the Upper and Lower San Fernando dams during the 1971 San Fernando earthquake (M 6.6) by using a finite-element model to estimate the strain potential at each node based on cyclic laboratory shear tests of soil samples. Lee (1974) and Serff et al. (1976) used the strain-potential method to model the reduction in the stiffness of soils and thus the permanent slope deformation. More recently, nonlinear inelastic soil models have been developed and implemented in 2-D and 3-D models (e.g., Prevost, 1981; Griffiths and Prevost, 1988; Elgamal et al., 1990).

All of these studies use highly complex models that, to be worthwhile, require a high density of high-quality data and sophisticated soil-constitutive models to predict the stress-strain behavior of the soils. The more advanced 3-D models are also quite computationally intensive. For this reason, stress-deformation modeling is generally practical only for critical projects such as earth dams and slopes affecting critical lifelines or structures. Stress-deformation analysis is innately site-specific and cannot be applied to regional problems. Even as a site-specific approach, it is generally reserved for critical projects because of the difficulty in procuring the needed data and the time and effort involved in the modeling procedure.

The advantage of stress-deformation modeling is that it gives the most accurate picture of what actually happens in the slope during an earthquake. Clearly, models that account for the complexity of spatial variability of properties and the stress-strain behavior of slope materials yield more reliable results. But stress-deformation modeling also has drawbacks. The complex modeling is warranted only if the quantity and quality of the data merit it. These modeling procedures can be quite challenging because of (1) the data acquisition required, commonly including undisturbed soil sampling and extensive laboratory testing of samples; (2) the need to select a suite of input ground motions; (3) the need for an accurate, nonlinear, stress-dependent, cyclic model of soil behavior; and (4) computational requirements. One last challenge of stress-deformation modeling is that most procedures model only distributed deformation, and the amount of deformation is limited. A very fine mesh is required to capture local deformation for landslides that move along a discrete basal failure surface.

4. Permanent-displacement analysis

In his 1965 Rankine lecture, Nathan Newmark (1965) introduced a method to assess the performance of slopes during earthquakes that bridges the gap between overly simplistic pseudostatic analysis and overly complex stress-deformation analysis. Newmark's method models a landslide as a rigid block that slides on an inclined plane; the block has a known yield or critical acceleration, the acceleration required to overcome basal resistance and initiate sliding (Figure 3). An earthquake strong-motion record of interest is selected, and those parts of the record that exceed the critical acceleration are integrated to obtain the velocity-time history of the block; the velocity-time history is then integrated to obtain the cumulative displacement of the landslide block (Figure 4). The user must judge the significance of the displacement.

The critical acceleration (a_c), in its simplest form, can be estimated as

$$a_c = (FS-1)g\sin\alpha, \tag{3}$$

where a_c is in terms of g, the acceleration of gravity, FS is the static factor of safety (the ratio of resisting to driving forces or moments in a slope), and α is the angle from the horizontal of the sliding surface (Newmark, 1965; Jibson, 1993). Eq. (3) assumes that the seismic force is applied parallel to the slope. Critical acceleration also can be estimated by iteratively performing a pseudostatic analyses to determine the yield coefficient, which, multiplied by g, is equivalent to the critical acceleration.

A key assumption of Newmark's method is that it treats a landslide as a rigid-plastic body: the mass does not deform internally, experiences no permanent displacement at accelerations below the critical level, and deforms plastically at constant stress along a discrete basal shear surface when the critical acceleration is exceeded. Other limiting assumptions commonly are imposed for simplicity:

1. The static and dynamic shearing resistance of the soil are taken to be the same (Newmark, 1965; Chang et al., 1984).
2. The critical acceleration is not strain dependent and thus remains constant throughout the analysis (Newmark, 1965; Makdisi and Seed, 1978; Chang et al., 1984; Ambraseys and Menu, 1988).
3. The upslope resistance to sliding is taken to be infinitely large such that upslope displacement is prohibited (Newmark, 1965; Chang et al., 1984; Ambraseys and Menu, 1988).
4. The effects of dynamic pore pressure are neglected. This assumption generally is valid for compacted or overconsolidated clays and very dense or dry sands (Newmark, 1965; Makdisi and Seed, 1978).

The first three of these assumptions were made early on to simplify calculations; more recent applications of Newmark's method do not require these assumptions. For example, Jibson and Jibson (2003) allow specifying strain-dependent critical acceleration as well as bi-lateral displacement, and dynamic strength testing can be performed to determine dynamic shear strengths. The assumption about dynamic pore pressure, however, is an important constraint on the analysis. There is no direct way to model dynamic pore-pressure in the Newmark analysis, and so this analysis should be applied with

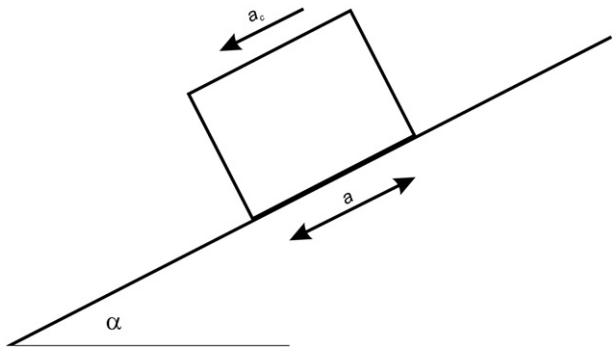

Fig. 3. Conceptual model of a rigid sliding-block analysis. The earthquake ground acceleration at the base of the block is denoted by a, a_c is the critical acceleration of the block, and α is the angle of inclination of the sliding surface.

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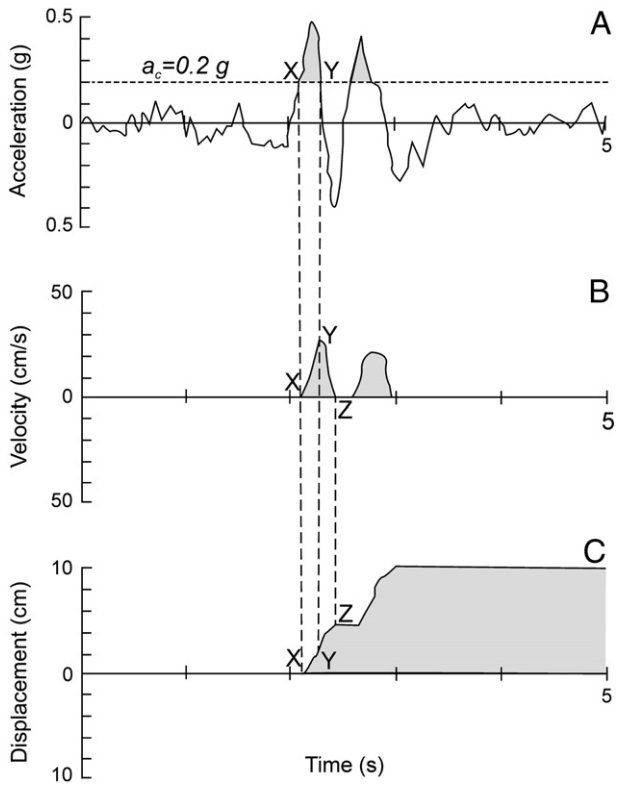

Fig. 4. Illustration of the Newmark integration algorithm, adapted from Wilson and Keefer (1983). A, earthquake acceleration-time history with critical acceleration (dashed line) of 0.2 g superimposed; B, velocity of the landslide versus time; C, displacement of landslide versus time. Points *X*, *Y*, and *Z* are for reference between plots.

great caution (or not at all) in situations where significant dynamic pore-pressures can develop.

Laboratory model tests (Goodman and Seed, 1966; Wartman et al., 2003, 2005) and analyses of earthquake-induced landslides in natural slopes (Wilson and Keefer, 1983) confirm that Newmark's method can fairly accurately predict slope displacements if slope geometry, soil properties, and earthquake ground motions are known. Newmark's method is simple to apply and provides an estimate of the inertial landslide displacement resulting from a given earthquake strong-motion record. Newmark's method has given rise to a family of analyses commonly referred to as permanent-displacement analysis.

4.1. Types of permanent-displacement analysis

Several variations of Newmark's method have been proposed that are designed to yield more accurate estimates of slope displacement by modeling the dynamic slope response more rigorously. This, of course, involves a trade-off. One great advantage of Newmark's method is its theoretical and practical simplicity. This simplicity, however, is the result of many assumptions that limit the accuracy of the results in many cases. The more sophisticated methods do a better job of modeling the dynamic response of the landslide material and thus yield more accurate displacement estimates, but again, there is a trade-off in the complexity of the analysis and the difficulty in acquiring the needed input parameters.

At present, analytical procedures for estimating permanent co-seismic landslide displacements can be grouped into three types: rigid-block, decoupled, and coupled.

4.1.1. Rigid-block analysis

Newmark's (1965) original methodology is generally referred to as rigid-block analysis. To briefly reiterate, the potential landslide block is modeled as a rigid mass that slides in a perfectly plastic manner on an inclined plane (see Figure 3). The mass experiences no permanent displacement until the base acceleration exceeds the critical acceleration of the block, at which time the block begins to move downslope. Displacements are calculated by integrating the parts of an acceleration-time history that lie above the critical acceleration to determine a velocity-time history. The velocity-time history is then integrated to yield the cumulative displacement (see Figure 4). Sliding continues until the relative velocity between the block and base reaches zero.

4.1.2. Decoupled analysis

Soon after Newmark published his rigid-block method, more sophisticated analyses were developed to account for the fact that landslide masses are not rigid bodies but deform internally when subjected to seismic shaking (Seed and Martin, 1966; Lin and Whitman, 1983). The most commonly used of such analyses was developed by Makdisi and Seed (1978), who produced design charts for estimating co-seismic displacements as a function of slope geometry, earthquake magnitude, and the ratio of yield acceleration to peak acceleration. This approach was calibrated to earth dams using a small number of strong-motion records.

A rigorous decoupled analysis estimates the effect of dynamic response on permanent sliding in a two-step procedure: (1) A dynamic-response analysis of the slope, assuming no failure surface, is performed using programs such as QUAD4M or SHAKE; by estimating the acceleration-time histories at several points within the slope, an average acceleration-time history for the slope mass above the potential failure surface is developed. The average acceleration has been referred to variously as k (Makdisi and Seed, 1978) or HEA, the horizontal equivalent acceleration (Bray and Rathje, 1998); peak values are generally referred to as k_{max} or MHEA, the maximum horizontal equivalent acceleration. The site-response analysis requires specification of the shear-wave velocity of the material, the thickness of the potential landslide, and the damping; for an equivalent-linear analysis, modulus-reduction and damping curves are also required. (2) The resulting time history is input into a rigid-block analysis, and the permanent displacement is estimated. This approach is referred to as a decoupled analysis because the computation of the dynamic response and the plastic displacement are performed independently. Decoupled analysis thus does not account for the effects of sliding displacement on the ground motion. Studies of actual seismically triggered landslides have shown that decoupled analysis can accurately predict field behavior (Pradel et al., 2005).

The method of Makdisi and Seed (1978) is the mostly widely used decoupled analysis. They provided design charts that allow estimation of displacement as a function of critical acceleration, ground motion, and earthquake magnitude. This approach has some limitations, however: (1) An estimate of the ground acceleration at the ground surface or crest of the dam is required, and there is significant uncertainty in this calculation. (2) The design charts show broad ranges of possible displacements that can span an order of magnitude, which leaves the user to judge where in this range to select a design displacement. (3) The analysis was designed and calibrated for earth dams, and yet it commonly has been applied to a wide range of situations, including applications in natural slopes, where some assumptions of the analysis are not valid. Bray and Rathje (1998) updated the Makdisi and Seed (1978) analysis for application to deeper sliding masses typical of landfills.

4.1.3. Coupled analysis

In a fully coupled analysis, the dynamic response of the sliding mass and the permanent displacement are modeled together so that the effect of plastic sliding displacement on the ground motions is taken into account. Lin and Whitman (1983) pointed out that the assumptions of the decoupled analysis introduce errors in the estimation of total slip and compared results for coupled and

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decoupled analyses. They showed that, in general, decoupled analysis yielded conservative results that were within about 20% of the coupled results. More recently, Rathje and Bray (1999, 2000) compared results from rigid-block analysis with linear and non-linear coupled and decoupled analyses.

Coupled analysis is the most sophisticated form of sliding-block analysis and also the most computationally intensive. In addition to the critical acceleration, other required inputs include the shear-wave velocities of the materials above and below the slip surface, the damping ratio, and the thickness of the potential landslide. Either linear-elastic or equivalent-linear soil models can be used for the computations; for equivalent-linear analysis, damping and modulus-reduction curves are needed. The largest impediment to the application of coupled analysis is the lack of published software packages for its implementation, but such a package is near completion (Jibson et al., in press).

Bray and Travasarou (2007) developed a simplified approach that used a nonlinear, fully coupled sliding-block model to produce a semi-empirical relationship for predicting displacement. The model requires specification of the yield acceleration, the fundamental period of the sliding mass (T_s), and the spectral acceleration of the ground motion at a period of $1.5T_s$.

4.2. Interpreting modeled displacements

The significance of modeled displacements must be judged by their probable effect on a potential landslide. Wieczorek et al. (1985) used 5 cm as the critical displacement leading to macroscopic ground cracking and failure of landslides in San Mateo County, California; Keefer and Wilson (1989) used 10 cm as the critical displacement for coherent landslides in southern California; and Jibson and Keefer (1993) used this 5–10 cm range as the critical displacement for landslides in the Mississippi Valley. In most soils, displacements in this range cause ground cracking, and previously undeformed soils can end up in a weakened or residual-strength condition. In such a case, static stability analysis in residual-strength conditions can be performed to determine the stability after earthquake shaking ceases (Jibson and Keefer, 1993).

Blake et al. (2002) made the following recommendations for application of rigid-block analysis in southern California:

- For slip surfaces intersecting stiff improvements (buildings, pools, etc.), median Newmark displacements should be less than 5 cm.
- For slip surfaces occurring in ductile (non-strain-softening) soil that do not intersect engineered improvements (landscaped areas, patios, etc.), median Newmark displacements should be less than 15 cm.
- In soils having significant strain softening (sensitivity>2), if the critical acceleration was calculated from peak shear strengths, displacements as large as 15 cm could trigger strength reductions, which in turn could destabilize the slope. For such cases, the design should be performed using either residual strengths and allowing median displacements less than 15 cm, or using peak strengths and allowing median displacements less than 5 cm.

The California Geological Survey's (2008) guidelines for mitigating seismic hazards state that displacements of 0–15 cm are unlikely to correspond to serious landslide movement and damage; displacements of 15–100 cm could be serious enough to cause strength loss and continuing failure; and displacements greater than 100 cm are very likely to correspond to damaging landslide movement. It should be noted here that these displacement thresholds pertain principally to deeper landslides; smaller, shallow landslides commonly are triggered at much lower displacement levels, perhaps 2–15 cm (Jibson et al., 2000).

Jibson and Michael (2009) used a similar range of Newmark displacements to define hazard categories for shallow landsliding on seismic landslide hazard maps of Anchorage, Alaska: 0–1 cm (low), 1–5 cm (moderate), 5–15 cm (high), >15 cm (very high).

Any level of critical displacement can be used according to the parameters of the problem under study and the characteristics of the landslide material. Ductile materials might be able to accommodate more displacement without general failure; brittle materials might accommodate less displacement. What constitutes “failure” differs according to the needs of the user. Results of laboratory shear-strength tests can be interpreted to estimate the strain necessary to reach residual strength.

Predicted displacements do not necessarily correspond directly to measurable slope movements in the field; rather, modeled displacements provide an index to correlate with field performance (Jibson et al., 1998, 2000; Rathje and Bray, 2000). For the sliding-block method to be useful in a rigorous predictive sense, modeled displacements must be quantitatively correlated with field performance. Pradel et al. (2005) conducted such a study on a single landslide following the 1994 Northridge, California earthquake and found good agreement between the predicted and observed displacements. Jibson et al. (1998, 2000) compared the inventory of all landslides triggered by the Northridge earthquake with predicted Newmark displacements. The results were then regressed using a Weibull model, which yielded the following equation (Jibson et al., 2000):

$$P(f) = 0.335\left[1-\exp\left(-0.048D_n^{1.565}\right)\right], \tag{4}$$

where $P(f)$ is probability of failure and D_n is Newmark displacement in centimeters. Eq. (4) can be used in any ground-shaking conditions to predict probability of slope failure as a function of Newmark displacement. This model was calibrated at regional scale using data from southern California, which included primarily shallow falls and slides in brittle rock and debris, and so it is only rigorously applicable to these types of landslides. Assigning geotechnical properties on a regional scale introduces significant uncertainty to such a model, which accounts for the upper bound probability of only 33.5%; calibration at a site-specific scale with more accurate geotechnical characterization would likely yield larger probability estimates.

4.3. Selecting an analysis

Reliable estimation of displacement depends on selecting the appropriate analysis. The best basis for this selection appears to be the period ratio, T_s/T_m, the ratio of the fundamental site period (T_s) to the mean period of the earthquake motion (T_m) (Rathje and Bray, 1999, 2000). T_s can be estimated as

$$T_s = 4H/V_s, \tag{5}$$

where H is the maximum vertical distance between the ground surface and slip surface used to estimate the critical acceleration, and V_s is the shear-wave velocity of the material above the slip surface. The mean period of the earthquake motion is defined as the average period weighted by the Fourier amplitude coefficients over a frequency range of 0.25–20 Hz (Rathje et al., 1998, 2004). Mean period (T_m, in seconds) can be estimated for shallow crustal earthquakes (data from western North America), rock site conditions, and no forward directivity as a function of earthquake moment magnitude (M_w) and source distance (r, in kilometers) as follows (Rathje et al., 2004):

$$\ln(T_m) = -1.00 + 0.18(M_w-6) + 0.0038r \text{ for } M_w \le 7.25 \tag{6a}$$

$$\ln(T_m) = -0.775 + 0.0038r \text{ for } M_w > 7.25. \tag{6b}$$

Theoretically, coupled analysis should yield the best results because it accounts for more of the complexities of the problem

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than either of the other analyses. In practice, however, coupled analysis can become numerically unstable for period ratios below about 0.1, where landslide masses behave rigidly. Therefore, a general guideline for selecting a sliding-block analysis would be to use rigid-block analysis for period ratios less than 0.1 and coupled analysis for period ratios greater than 0.1.

As coupled analysis becomes more accessible, little justification will exist to continue use of decoupled analysis. Perhaps, in light of the long historical use of decoupled analysis, one appropriate use would be for comparison with past applications. Thus, use of decoupled analysis will probably continue for some time.

In situations where coupled analysis is unavailable to a user, selecting between decoupled and rigid-block analysis requires some judgment. Table 2 provides guidelines, in the terms of a comparison with results from coupled analysis. Rigid-block analysis is appropriate for analyzing thin, stiff landslides having a period ratio of 0.1 or less. Between 0.1 and 1, rigid-block analysis yields unconservative results and should not be used; decoupled analysis yields conservative to very conservative results in this range. For period ratios between 1 and 2, rigid-block analysis yields conservative results, but results from decoupled analysis are closer to those from coupled analysis. For period ratios greater than about 2, rigid-block analysis tends to yield highly over-conservative results that significantly overestimate displacement, and decoupled analysis can be either conservative or unconservative; therefore, in this range coupled analysis should be used.

4.4. Applications of permanent-displacement analysis

Permanent-displacement analysis has been applied in a variety of ways to slope-stability problems. Most early applications dealt with the seismic performance of dams and embankments (e.g., Seed et al., 1973; Makdisi and Seed, 1978; Seed, 1979; Yegian et al., 1991). Permanent-displacement analysis also has been successfully applied to landslides in natural slopes (Wilson and Keefer, 1983; Pradel et al., 2005), and it is being used increasingly in slope design, solid-waste landfills, and other engineered works (e.g., Bray and Rathje, 1998; Blake et al., 2002).

Several simplified approaches have been proposed for applying permanent-displacement analysis; these involve developing empirical relationships to predict slope displacement as a function of critical acceleration and one or more measures of earthquake shaking. Many such studies plot displacement against critical acceleration ratio-the ratio of critical acceleration to peak ground acceleration (Franklin and Chang, 1977; Makdisi and Seed, 1978; Ambraseys and Menu, 1988; Jibson, 2007; Saygili and Rathje, 2008; Rathje and Saygili, 2009). Other studies have related critical acceleration ratio to some normalized form of displacement (Lin and Whitman, 1983; Yegian et al., 1991). Jibson (1993, 2007) and Jibson et al. (1998, 2000) related Newmark displacement to critical acceleration and Arias (1970) intensity (I_a), the integral over time of the squared accelerations in a strong-motion record. As noted previously, Bray and Travasarou (2007) developed a simplified method for fully coupled analysis that predicts displacement as a function of yield acceleration, site period, and spectral acceleration. Miles and Ho (1999) and Miles and Keefer (2000) compared results from these simplified methods with their own method of rigorously integrating artificially generated strong-motion time histories.

The commonest application of these simplified methods is in making seismic landslide hazard maps, where regional estimation of Newmark displacement in a GIS model is required. Wieczorek et al. (1985) were the first to use Newmark analysis as a basis for seismic landslide microzonation, and methods for such applications have evolved steadily since that first study (e.g., Jibson et al., 1998, 2000; Mankelow and Murphy, 1998; Luzi and Pergalani, 1999; Miles and Ho, 1999; Miles and Keefer, 2000, 2001; Del Gaudio et al., 2003; Rathje and Saygili, 2008, 2009; Jibson and Michael, 2009). Such applications generally involve GIS modeling in which study areas are gridded, and discrete estimates of coseismic displacement are generated for each grid cell using simplified regression equations.

Table 2
Guidelines for selecting appropriate sliding-block analysis.

Slide type	T_s/T_m	Rigid-block analysis	Decoupled analysis
Stiffer, thinner slides	0–0.1	Best results	Good results
↓	0.1–1.0	Unconservative	Conservative to very conservative
↓			
Softer, thicker slides	1.0–2.0	Conservative	Conservative
	>2.0	Very conservative	Conservative to unconservative

T_s is the site period.
T_m is the mean period of the earthquake shaking.

5. Discussion

So what is the current status of each of these types of analysis, and where and when should they be used? Pseudostatic analysis is still very widely used in practice and has a deep reservoir of engineering judgment behind it. It is conceptually simple, but the process of selecting a seismic coefficient commonly lacks a rational basis, and the analysis tends to be over-conservative. Even more troubling is that this analysis, which *should* be conservative, sometimes is unconservative and therefore must be calibrated to the specific conditions being modeled. This seldom happens in practice, however, because of a long-term sense of judgment regarding its application. The current availability of software that facilitates performing rigorous permanent-displacement analysis (Jibson and Jibson, 2003; Jibson et al., in press) renders the most powerful rationale for using pseudostatic analysis—its simplicity—invalid: performing permanent-displacement analysis is currently little more difficult than performing pseudostatic analysis. Therefore, the most appropriate applications for pseudostatic analysis are probably limited to preliminary analyses and screening procedures that precede a more sophisticated analysis (Stewart et al., 2003).

Stress-deformation analysis is still the gold standard for a single-site analysis where sufficient data exist to merit it. For critical facilities such as dams and for embankments or slopes adjacent to critical lifelines or structures, the cost and effort of stress-deformation analysis is generally justified.

Permanent-displacement analysis seems increasingly able to fill the gap between the two end members represented by the other two families of analysis. The development of decoupled and coupled analysis facilitates permanent-displacement analysis being applied to a variety of landslide types. Permanent-displacement analysis has two principle advantages over stress-deformation analysis: (1) it is much easier to use, and (2) it allows modeling of large displacements on discrete basal shear surfaces. Its main advantage over pseudostatic analysis is that it provides a quantitative measure of what happens after the critical acceleration is exceeded, the point at which a pseudostatic analysis simply defines “failure” as having occurred. Permanent-displacement analysis begins, in fact, exactly where pseudostatic analysis ends: at the moment the critical or yield acceleration is exceeded. The displacement thus modeled provides a quantitative measure—an index—of co-seismic slope performance. This index can then be used in a variety of ways, including defining hazard categories for predictive maps (e.g., Wieczorek, et al., 1985; Jibson and Michael, 2009), predicting actual field displacement (e.g., Blake et al., 2002; Pradel et al., 2005), or predicting the probability of slope failure through calibration using inventories of landslides triggered during actual earthquakes (Jibson et al., 1998, 2000).

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Newmark's (1965) method and its subsequent variations were developed to analyze the seismic behavior of earth dams and embankments (e.g., Franklin and Chang, 1977; Makdisi and Seed, 1978; Seed, 1979; Lin and Whitman, 1983). These large earth structures commonly have well-defined, homogeneous properties; are constructed largely of relatively ductile fine-grained materials; and are principally subject to deep modes of failure. These properties of large earth structures were the very reason that coupled and decoupled analysis were later developed, to better model engineered earth structures by taking into account the dynamic deformation of the soil mass and the effects of coseismic displacement on the response of the slide mass (Bray and Rathje, 1998; Rathje and Bray, 1999, 2000). These and other studies (e.g., Kramer and Smith, 1997; Wartman et al., 2003, 2005; Lin and Wang, 2006) make clear that traditional Newmark (rigid-block) analysis does not yield acceptable results for the seismic performance of large engineered earth structures. Wartman et al. (2003, 2005) and Rathje and Bray (1999, 2000) provided detailed treatments of specific combinations of site and shaking conditions that are and are not adequately modeled by rigid-block analysis.

Rigid-block analysis is best suited to a very different type of slope failure: earthquake-triggered landslides in natural slopes, an application first proposed by Wilson and Keefer (1983). As stated previously, rigid-block analysis is applicable to thinner landslides in stiffer material ($T_s/T_m<0.1$); in practical terms, this means fairly shallow slides and falls in rock and debris. Keefer's (1984, 2002) analysis of data from worldwide earthquakes indicated that the large majority of earthquake-triggered landslides are of this type. Documentations of landslides from several earthquakes have indicated that such failures commonly make up 90% or more of triggered landslides (e.g., Harp et al., 1981; Harp and Jibson, 1995, 1996; Keefer and Manson, 1998; Jibson et al., 2004, 2006). These types of landslides are well-suited to rigid-block analysis because the brittle surficial material behaves rigidly, and the relatively thin landslide masses do not experience significant site response that would modify the incident ground motions. Thus, the large majority of seismically triggered landslides in natural slopes are shallow landslides in fairly brittle surface material and so are modeled well by rigid-block analysis. Larger, deeper landslides, which are much less common but potentially more destructive where they do occur, should be modeled using coupled analysis.

So where do we go from here? The discontinuity between analytical methods that work for thinner landslides in stiffer materials (rigid-block analysis) versus thicker landslides in softer materials (coupled analysis) presents a challenge: where exactly is the boundary between these two types of slides? A T_s/T_m ratio of 0.1 has been proposed for this boundary, but the transition clearly is not that abrupt in nature. A robust analytical method that accommodates a full range of period ratios would be ideal, but it might be elusive for simple mathematical reasons. Rathje and Antonakos (this volume) have proposed a simplified empirical method that bridges this divide and is applicable across the full range of period ratios.

6. Conclusion

The three families of analyses for assessing seismic slope stability each have their appropriate application. Pseudostatic analysis, because of its crude characterization of the physical process, should be used only for preliminary or screening analyses. Stress-deformation analysis is best suited to large earth structures such as dams and embankments; it is too complex and expensive for more routine applications. Permanent-displacement analysis provides a valuable middle ground between these end members. It is simple to apply and provides far more information than does pseudostatic analysis. Rigid-block analysis is suitable for thinner, stiffer landslides, which typically comprise the large majority of earthquake-triggered landslides. Coupled analysis is appropriate for deeper landslides in softer material, which could include large earth structures and deep landslides. Modeled displacements provide a useful index to seismic slope performance and must be interpreted using judgment and according to the parameters of the investigation.

References

Ambraseys, N.N., Menu, J.M., 1988. Earthquake-induced ground displacements. Earthquake Engineering and Structural Dynamics 16, 985–1006.

Arias, A., 1970. A measure of earthquake intensity. In: Hansen, R.J. (Ed.), Seismic Design for Nuclear Power Plants. Massachusetts Institute of Technology Press, Cambridge, MA, pp. 438–483.

Blake, T.F., Hollingsworth, R.A., Stewart, J.P., 2002. Recommended procedures for implementation of DMG special publication 117—guidelines for analyzing and mitigating landslide hazards in California. Southern California Earthquake Center, Los Angeles, CA. 127 pp.

Bray, J.D., Rathje, E.M., 1998. Earthquake-induced displacements of solid-waste landfills. Journal of Geotechnical and Geoenvironmental Engineering 124, 242–253.

Bray, J.D., Travasarou, T., 2007. Simplified procedure for estimating earthquake-induced deviatoric slope displacements. Journal of Geotechnical and Geoenvironmental Engineering 133, 381–392.

Bray, J.D., Travasarou, T., 2009. Pseudostatic coefficient for use in simplified seismic slope stability evaluation. Journal of Geotechnical and Geoenvironmental Engineering 135, 1336–1340.

California Division of Mines and Geology, 1997. Guidelines for Evaluating and Mitigating Seismic Hazards in California. California Division of Mines and Geology Special Publication 117. 74 pp.

California Geological Survey, 2008. Guidelines for Evaluating and Mitigating Seismic Hazards in California. California Geological Survey Special Publication 117A. 98 pp.

Chang, C.J., Chen, W.F., Yao, J.T.P., 1984. Seismic displacements in slopes by limit analysis. Journal of Geotechnical Engineering 110, 860–874.

Clough, R.W., 1960. The finite element method in plane stress analysis. Proceedings of the 2nd Conference on Electronic Computation. American Society of Civil Engineers, Structural Division, Pittsburgh, PA. Sept. 1960.

Clough, R.W., Chopra, A.K., 1966. Earthquake stress analysis in earth dams. ASCE Journal of the Engineering Mechanics Division 92, 197–211.

Courant, R., 1943. Variational methods for solution of equilibrium and vibration. Bulletin of the American Mathematical Society 49, 1–43.

Del Gaudio, V., Pierri, P., Wasowski, J., 2003. An approach to time-probabilistic evaluation of seismically induced landslide hazard. Seismological Society of America Bulletin 93, 557–569.

Elgamal, A.M., Scott, R.F., Succarieh, M.F., Yan, L.P., 1990. La Villita dam response during five earthquakes including permanent deformation. Journal of Geotechnical Engineering 116, 1443–1462.

Franklin, A.G., Chang, F.K., 1977. Earthquake resistance of earth and rock-fill dams: U.S. Army Corps of Engineers Waterways Experiment Station. Miscellaneous Paper S-71-17, 59 pp.

Goodman, R.E., Seed, H.B., 1966. Earthquake-induced displacements in sand embankments. Journal of the Soil Mechanics and Foundations Division 92, 125–146.

Griffiths, D.V., Prevost, J.H., 1988. Two- and three-dimensional finite element analyses of the Long Valley Dam. Technical Report NCEER-88-0015. National Center for Earthquake Engineering Research, Buffalo, NY.

Harp, E.L., Jibson, R.W., 1995. Inventory of landslides triggered by the Northridge, California earthquakes. U.S. Geological Survey Open-File Report 95–213. 17 pp.

Harp, E.L., Jibson, R.W., 1996. Landslides triggered by the 1994 Northridge, California earthquake. Seismological Society of America 86 (1B), S319–S332.

Harp, E.L., Wilson, R.C., Wieczorek, G.F., 1981. Landslides from the February 4, 1976 Guatemala earthquake. U.S. Geological Survey Professional Paper 1204-A. 35 pp.

Hynes-Griffin, M.E., Franklin, A.G., 1984. Rationalizing the seismic coefficient method. U.S. Army Corps of Engineers Waterways Experiment Station. Miscellaneous Paper GL-84-13, 37 pp.

Jibson, R.W., 1993. Predicting earthquake-induced landslide displacements using Newmark's sliding block analysis. Transportation Research Record 1411, 9–17.

Jibson, R.W., 2007. Regression models for estimating coseismic landslide displacement. Engineering Geology 91, 209–218.

Jibson, R.W., Jibson, M.W., 2003. Java programs for using Newmark's method and simplified decoupled analysis to model slope performance during earthquakes. U.S. Geological Survey Open-File Report 03-005. version 1.1.

Jibson, R.W., Keefer, D.K., 1993. Analysis of the seismic origin of landslides: examples from the New Madrid seismic zone. Geological Society of America Bulletin 105, 521–536.

Jibson, R.W., Michael, J.A., 2009. Maps showing seismic landslide hazards in Anchorage, Alaska. U.S. Geological Survey Scientific Investigations Map 3077. 2 sheets (scale 1:25, 000), 11-p. pamphlet.

Jibson, R.W., Harp, E.L., Michael, J.A., 1998. A method for producing digital probabilistic seismic landslide hazard maps—an example from the Los Angeles, California, area. U.S. Geological Survey Open-File Report 98–113. 17 p., 2 pl.

Jibson, R.W., Harp, E.L., Michael, J.A., 2000. A method for producing digital probabilistic seismic landslide hazard maps. Engineering Geology 58, 271–289.

Jibson, R.W., Harp, E.L., Schulz, W., Keefer, D.K., 2004. Landslides triggered by the 2002 M-7.9 Denali Fault, Alaska, earthquake and the inferred nature of the strong shaking. Earthquake Spectra 20, 669–691.

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Jibson, R.W., Harp, E.L., Schulz, W., Keefer, D.K., 2006. Large rock avalanches triggered by the M-7.9 Denali Fault, Alaska, earthquake of 3 November 2002. Engineering Geology 83, 144–160.
Jibson, R.W., Rathje, E.M., Jibson, M.W., Lee, Y.W., in press, SLAMMER—Seismic LAndslide Movement Modeled using Earthquake Records. U.S. Geological Survey Techniques and Methods.
Keefer, D.K., 1984. Landslides caused by earthquakes. Geological Society of America Bulletin 95, 406–421.
Keefer, D.K., 2002. Investigating landslide caused by earthquakes—a historical review. Surveys in Geophysics 23, 473–510.
Keefer, D.K., Manson, M.W., 1998. Regional distribution and characteristics of landslides generated by the earthquake. In: Keefer, D.K. (Ed.), The Loma Prieta, California, earthquake of October 17, 1989—landslides: U.S. Geological Survey Professional Paper 1551-C, pp. C7–C32.
Keefer, D.K., Wilson, R.C., 1989. Predicting earthquake-induced landslides, with emphasis on arid and semi-arid environments. Landslides in a Semi-arid Environment, 2. Inland Geological Society, Riverside, CA, pp. 118–149.
Kramer, S.L., 1996. Geotechnical Earthquake Engineering. Prentice Hall, Upper Saddle River, NJ. 653 pp.
Kramer, S.L., Smith, M.W., 1997. Modified Newmark model for seismic displacements of compliant slopes. Journal of Geotechnical and Geoenvironmental Engineering 123, 635–644.
Lee, K.L., 1974. Seismic permanent deformation in earth dams. Report No. UCLA-ENG-7497. School of Engineering and Applied Science. University of California, Los Angeles, CA.
Lin, M.L., Wang, K.L., 2006. Seismic slope behavior in a large-scale shaking table model test. Engineering Geology 86, 118–133.
Lin, J.S., Whitman, R.V., 1983. Earthquake induced displacements of sliding blocks. Journal of Geotechnical Engineering 112, 44–59.
Luzi, L., Pergalani, F., 1999. Slope instability in static and dynamic conditions for urban planning: the "Oltre Po Pavese" case history (Regione Lombardia – Italy). Natural Hazards 20, 57–82.
Makdisi, F.I., Seed, H.B., 1978. Simplified procedure for estimating dam and embankment earthquake-induced deformations. ASCE Journal of the Geotechnical Engineering Division 104, 849–867.
Mankelow, J.M., Murphy, W., 1998. Using GIS in the probabilistic assessment of earthquake triggered landslide hazards. Journal of Earthquake Engineering 2, 593–623.
Marcuson, W.F., 1981. Moderator's report for session on Earth Dams and Stability of Slopes under Dynamic Loads. Proceedings, International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, St. Louis, MO, 3, p. 1175.
Miles, S.B., Ho, C.L., 1999. Rigorous landslide hazard zonation using Newmark's method and stochastic ground motion simulation. Soil Dynamics and Earthquake Engineering 18, 305–323.
Miles, S.B., Keefer, D.K., 2000. Evaluation of seismic slope-performance models using a regional case study. Environmental and Engineering Geoscience 6, 25–39.
Miles, S.B., Keefer, D.K., 2001. Seismic Landslide Hazard for the City of Berkeley, California. U.S. Geological Survey Miscellaneous Field Studies Map MF-2378.
Newmark, N.M., 1965. Effects of earthquakes on dams and embankments. Geotechnique 15, 139–159.
Pradel, D., Smith, P.M., Stewart, J.P., Raad, G., 2005. Case history of landslide movement during the Northridge earthquake. Journal of Geotechnical and Geoenvironmental Engineering 131, 1360–1369.
Prevost, J.H., 1981. DYNAFLOW: a nonlinear transient finite element analysis program. Department of Civil Engineering, Princeton University, Princeton, NJ.
Rathje, E.M., Bray, J.D., 1999. An examination of simplified earthquake-induced displacement procedures for earth structures. Canadian Geotechnical Journal 36, 72–87.
Rathje, E.M., Bray, J.D., 2000. Nonlinear coupled seismic sliding analysis of earth structures. Journal of Geotechnical and Geoenvironmental Engineering 126, 1002–1014.
Rathje, E.M., Saygili, G., 2008. Probabilistic seismic hazard analysis for the sliding displacement of slopes: scalar and vector approaches. Journal of Geotechnical and Geoenvironmental Engineering 134, 804–814.
Rathje, E.M., Saygili, G., 2009. Probabilistic assessment of earthquake-induced sliding displacements of natural slopes. Bulletin of the New Zealand Society of Earthquake Engineering 41, 18–27.
Rathje, E.M., Abrahamson, N.A., Bray, J.D., 1998. Simplified frequency content estimates of earthquake ground motions. Journal of Geotechnical Engineering 124, 150–159.
Rathje, E.M., Faraj, F., Russell, S., Bray, J.D., 2004. Empirical relationships for frequency content parameters of earthquake ground motions. Earthquake Spectra 20, 119–144.
Saygili, G., Rathje, E.M., 2008. Empirical predictive models for earthquake-induced sliding displacements of slopes. Journal of Geotechnical and Geoenvironmental Engineering 134, 790–803.
Seed, H.B., 1979. Considerations in the earthquake-resistant design of earth and rockfill dams. Geotechnique 29, 215–263.
Seed, H.B., Martin, G.R., 1966. The seismic coefficient in earth dam design. ASCE Journal of the Soil Mechanics and Foundations Division 92, 25–58.
Seed, H.B., Lee, K.L., Idriss, I.M., Makdisi, R., 1973. Analysis of the slides in the San Fernando dams during the earthquake of Feb. 9, 1971. Report No. EERC 73-2. Earthquake Engineering Research Center, University of California, Berkeley. 150 pp.
Serff, N., Seed, H.B., Makdisi, F.I., Chang, C.Y., 1976. Earthquake-induced deformations of earth dams. Report No. EERC 76-4. Earthquake Engineering Research Center, University of California, Berkeley. 140 pp.
Stewart, J.P., Blake, T.F., Hollingsworth, R.A., 2003. A screen analysis procedure for seismic slope stability. Earthquake Spectra 19, 697–712.
Terzhagi, K., 1950. Mechanism of landslides. In: Paige, S. (Ed.), Application of Geology to Engineering Practice (Berkey Volume). Geological Society of America, New York, NY, pp. 83–123.
Wartman, J., Bray, J.D., Seed, R.B., 2003. Inclined plane studies of the Newmark sliding block procedure. Journal of Geotechnical and Geoenvironmental Engineering 129, 673–684.
Wartman, J., Seed, R.B., Bray, J.D., 2005. Shaking table modeling of seismically induced deformations in slopes. Journal of Geotechnical and Geoenvironmental Engineering 131, 610–622.
Wieczorek, G.F., Wilson, R.C., Harp, E.L., 1985. Map showing slope stability during earthquakes in San Mateo County, California. U.S. Geological Survey Miscellaneous Investigations Map I-1257-E, scale 1:62,500.
Wilson, R.C., Keefer, D.K., 1983. Dynamic analysis of a slope failure from the 6 August 1979 Coyote Lake, California, earthquake. Bulletin. Seismological Society of America 73, 863–877.
Yegian, M.K., Marciano, E.A., Ghahraman, V.G., 1991. Earthquake-induced permanent deformations: probabilistic approach. Journal of Geotechnical Engineering 117, 35–50.