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Differential Equations Governing Slip-Induced Pore-Pressure Fluctuations in a Water-Saturated Granular Medium[1]

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Macroscopic frictional slip in water-saturated granular media occurs commonly during landsliding, surface faulting, and intense bedload transport. A mathematical model of dynamic pore-pressure fluctuations that accompany and influence such sliding is derived here by both inductive and deductive methods. The inductive derivation shows how the governing differential equations represent the physics of the steadily sliding array of cylindrical fiberglass rods investigated experimentally by Iverson and LaHusen (1989). The deductive derivation shows how the same equations result from a novel application of Biot's (1956) dynamic mixture theory to macroscopic deformation. The model consists of two linear differential equations and five initial and boundary conditions that govern solid displacements and pore-water pressures. Solid displacements and water pressures are strongly coupled, in part through a boundary condition that ensures mass conservation during irreversible pore deformation that occurs along the bumpy slip surface. Feedback between this deformation and the pore-pressure field may yield complex system responses. The dual derivations of the model help explicate key assumptions. For example, the model requires that the dimensionless parameter B*, defined here through normalization of Biot's equations, is much larger than one. This indicates that solid-fluid coupling forces are dominated by viscous rather than inertial effects. A tabulation of physical and kinematic variables for the rod-array experiments of Iverson and LaHusen and for various geologic phenomena shows that the model assumptions commonly are satisfied. A subsequent paper will describe model tests against experimental data.*

KEY WORDS: landslide, fault, mixture theory, mathematical model, pore pressure, dynamic poroelasticity.

INTRODUCTION

Dynamic pore-pressure fluctuations accompany rapid frictional slip in water-saturated granular media (Iverson and LaHusen, 1989; Eckersley, 1990; Ochiai *et al.*, 1991). By mediating effective stresses at grain contacts, the fluctuations can influence slip dynamics, perhaps leading to unstable or chaotic motion. Iverson and LaHusen (1989) measured such fluctuations in an array of cylin-

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drical fiberglass rods sheared steadily along a discrete, horizontal slip surface and in laboratory landslides comprising 40 m^3 of poorly sorted, sandy soil. This paper describes a physically based mathematical model that accurately predicts the magnitude, frequency spectrum, propagation speed, and attenuation of the fluctuations for conditions that match those of the rod-array experiments. A subsequent paper will describe normalization and numerical solution of the model equations as well as model tests against experimental data.

To reveal the connection between the forces that produce pore-pressure fluctuations and the mathematics that produce the model equations, two derivations are presented. The first derivation emphasizes the forces in the rod-array experiments of Iverson and LaHusen (1989). Although this inductive derivation employs general physical principles, such as Newton's laws of motion, and holds the advantage of being simple, concise, and closely linked to the experimental design, it also relies on *ad hoc* assumptions that may not widely apply. It lacks a formal connection to a broader theoretical framework. In contrast, the second derivation employs mathematical deduction to extract the model equations from the theoretical framework established by Biot (1956). It shows that Biot's formulation of Lagrange's equations can be applied to macroscopic, frictional slip as well as to more traditional, infinitesimal wave propagation problems. This deductive derivation holds the advantage of both formalism and rigor, but does so at the expense of mathematical detail that can obfuscate the physics. Importantly, however, it helps quantify the conditions under which the model assumptions apply. A tabulation of representative parameter values provides a basis for assessing model applicability to the rod-array experiments of Iverson and LaHusen (1989) and to geological phenomena such as landsliding and faulting.

INDUCTIVE DERIVATION

Consider a close-packed array of solid rods, each of which is approximately circular in cross-section and has mass density ρ_s and mean diameter δ (Fig. 1). The length of each rod and the breadth and width of the array are much larger than δ, so the array approximates one that is two-dimensional and infinite. The positions of all rods within the array are fixed with respect to one another, except along a horizontal slip surface that transects the array. A Cartesian coordinate system is fixed with respect to the part of the array below the slip surface. Relative to this coordinate system, the upper part of the array slides over the lower part with a constant horizontal velocity v_x. The vertical velocity component of the sliding rods v_y is free to fluctuate. Gravity acts in the negative y direction.

A Newtonian fluid of constant mass density ρ_f and viscosity μ fills the interstitial spaces between the rods in the array and along the slip surface. This

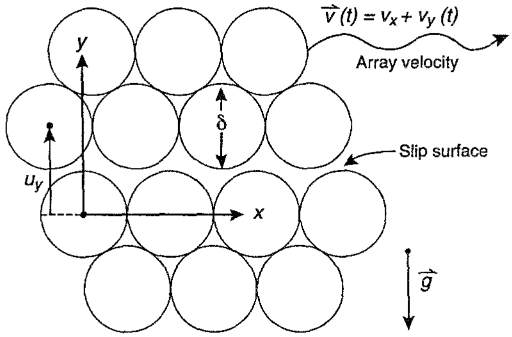

Fig. 1. Schematic vertical cross-section and coordinate system for a region near the slip surface of the water-saturated, sliding array of nearly cylindrical rods used in the experiments of Iverson and LaHusen (1989). The horizontal component of velocity v_x is constant but the vertical component v_y is a function of time.

pore fluid can flow through slit-like spaces between the rods, because the rods are imperfectly circular in cross-section and contact one another discontinuously. In a typical prototype, the pore fluid would be liquid water and the rods would be rock fragments, mineral grains, or soil aggregates.

The objective is to calculate both the transient fluid pressure field and the trajectory of the rods as they move along the slip surface. For sufficiently small sliding rates (small v_x), the fluid pressure field deviates insignificantly from hydrostatic, the rod layers adjacent to the slip surface remain in contact, and the sliding rods' trajectories are simply determined by the geometry of the slip surface. For sufficiently large sliding rates, the rods' trajectories may be influenced by the nonhydrostatic fluid pressure field that develops as a consequence of rod motion, and the sliding rods may intermittently lose contact with the underlying rods. Evaluation of the coupling between rod motion and fluid pressures provides a measure of sliding rates that are "small" and "large."

Motion of Solids

To determine the macroscopic trajectory of the sliding array of rods, I ignore its elastic deformation, an effect I will consider later. I then need only to calculate the vertical coordinate of the axis of a single sliding rod u_y as a function of time, because the sliding array translates irrotationally, and its horizontal velocity v_x is fixed. The coordinate, u_y identifies the axis of a sliding rod immediately above the slip surface and is measured with respect to an origin located on the axis of an arbitrary rod adjacent to and beneath the slip surface

(Fig. 1). Thus, for example, if the slip surface is dilated and the rod array is at rest, $u_y = \delta$.

Consider a unit cell of the rod array that extends vertically from the slip surface to the upper margin of the array (Fig. 2). The cell has width δ and length L, where L is measured along the rods' axes. The cell height is $(M - 1)(\sqrt{3}/2)\,\delta + \delta$, where M is the number of rods stacked atop the slip surface. For large values of M, $(M - 1)(\sqrt{3}/2)\,\delta + \delta \approx (\sqrt{3}/2)\,\delta M$, with an approximation error of about 1% for $M = 10$. Adopting this approximation, a simple geometric analysis shows that the mass of solids in the unit cell, m, is given by

$$m = \frac{\sqrt{3}}{2}\,\rho_s \delta^2 ML(1 - n) \tag{1}$$

where n is the porosity of the array. The geometry of a densely packed array of perfectly cylindrical rods yields $n = 1 - \pi/[2\sqrt{3} + (4 - 2\sqrt{3})/M]$, which reduces to $n = 1 - \pi/2\sqrt{3}$ as $M \rightarrow \infty$. These formulas yield $n \approx 0.1069$ for $M = 10$ and $n \approx 0.0931$ for $M \rightarrow \infty$. For the densely packed array of imperfectly cylindrical rods, I use the approximation $n = 0.1$.

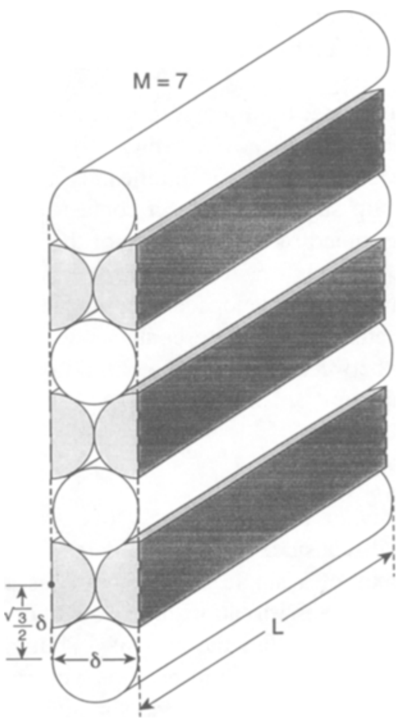

Fig. 2. Schematic cut-away view of a unit cell of the sliding rod array for a case with seven rod layers atop the slip surface. The distance from the center of a layer of rods to the center of an adjacent layer is $(\sqrt{3}/2)\ \delta$. In the mathematical model $L \gg \delta$.

Temporarily ignoring constraints due to the geometry of the slip surface, Newton's second law governs the vertical motion of solids in the unit cell as it undergoes steady horizontal sliding:

$$m \frac{d^2 u_y}{dt^2} = \Sigma F \tag{2}$$

where t is time and ΣF is the sum of all forces acting on the solids in the y direction. One such force is that due to gravitational acceleration, F_g. Another such force is that due to the instantaneous fluid pressure field, F_p. This force includes a hydrostatic (buoyancy) component and a non-equilibrium component that exists only if a non-hydrostatic fluid pressure gradient is present. Other forces that might be significant are those due to rod-to-rod friction and collisions along the slip surface. The frictional force can be evaluated easily, but it does not affect vertical motion because the constant horizontal velocity v_x is maintained by an external force. Collisional forces in this two-phase system may be quite complex and may involve both elastic and inelastic components (cf. Davis, 1986). However, only the elastic rebound of rods affects their vertical motion if v_x is fixed, and, as a first approximation, I assume that this rebound is negligible. Acceleration of the solid rods through the adjacent fluid is also subject to the virtual- or added-mass effect described by Batchelor (1967) and Biot (1956). Here I assume this effect is negligible; I assess this assumption in the deductive derivation to follow.

Explicit expressions for the gravity force and net fluid-pressure force acting on the solids in the unit cell are

$$F_g = -mg \tag{3}$$

$$\begin{aligned} F_p &= \frac{\rho_f}{\rho_s} mg - \delta L \int_{y = u_y - \delta/2}^{u_y - \delta/2 + (\sqrt{3}/2)M\delta} \frac{\partial p}{\partial y}\, dy \\ &= \frac{\rho_f}{\rho_s} mg - \delta L \left[p\left(u_y - \delta/2 + \frac{\sqrt{3}}{2} M\delta,\, t\right) - p(u_y - \delta/2,\, t) \right] \end{aligned} \tag{4}$$

in which g is the magnitude of gravitational acceleration, and $(\rho_f/\rho_s)\, mg$ is the hydrostatic or buoyancy component of the fluid-pressure force. The nonhydrostatic or non-equilibrium component of the fluid-pressure force is a function of both position and time, and it is given by the second term on the right-hand side of (4). This term represents the net force due to the difference in the non-equilibrium pore pressure p between the top and bottom of the unit cell (Fig. 2). The non-equilibrium fluid-pressure force is analogous to the so-called seepage force in quasistatic porous media (e.g., Iverson and Major, 1986), and it acts in a direction opposite to that of the non-equilibrium fluid pressure gradient.

Combination of (2), (3), and (4) yields an equation governing the vertical motion of the rod array

$$\frac{d^2u_y}{dt^2} = -\left(1 - \frac{\rho_f}{\rho_s}\right) g - \frac{\delta L}{m}\left[p\left(u_y - \delta/2 + \frac{\sqrt{3}}{2} M\delta, t\right) - p(u_y - \delta/2, t)\right] \tag{5}$$

Appropriate initial conditions to be used in solving (5) specify that the slip surface is dilated and the vertical velocity of the rods is zero:

$$u_y(0) = \delta \tag{6}$$

$$v_y(0) = \frac{du_y}{dt}(0) = 0 \tag{7}$$

These conditions apply at time $t = 0$, when sliding begins, but they must be modified periodically to obtain solutions for steady sliding, because the vertical motion during sliding is controlled, in part, by geometrical constraints that are not represented in (5).

Motion of Fluid

The pore-fluid pressure and flow fields depend on the vertical motion of the solid rods. Conceptually, I separate the fluid behavior into two parts: that which occurs in the irreversibly deforming space between the layers of rods that bound the slip surface, and that which occurs in the elastically deforming array of rods that translates intact. I model the fluid behavior in the translating array of rods as that in a homogeneous, isotropic, Darcian porous medium; I implicitly average the fluid flow and pressure fields over representative elementary volumes (Bear, 1972) that contain a number of rods and pores. I address this Darcian behavior first.

As the rod array moves, viscous drag causes the pore fluid to move with it, but the fluid also tends to move independently owing to its inertia and to boundary forcing caused by displacement of fluid along the slip surface. Because v_x is constant and the array is isotropic, the horizontal component of Darcian fluid motion in the array is steady and is driven only by the horizontal component of the gradient of p. Thus steady, horizontal, Darcian fluid motion is independent of transient, vertical, Darcian fluid motion, and horizontal flow does not affect vertical forces. Vertical fluid flow, however, is closely coupled to the vertical motion of the rod array, and I find it advantageous to evaluate this vertical fluid motion with respect to coordinates that translate with the array. I consequently define the moving vertical coordinate

$$y_m = y - u_y + \delta/2 \tag{8}$$

so that $y_m = 0$ is always satisfied at the base of the sliding array. If the fluid were static, p would equal 0 everywhere, even in this moving coordinate system.

When $v_y \neq 0$, fluid must move and the fluid pressure distribution in the array cannot be hydrostatic. The disequilibrium fluid pressure p propagates by two means: a fast mode similar to elastic compressional waves and a slow mode similar to quasistatic pore-pressure diffusion. Both modes of propagation are coupled to elastic deformation of the array, and both are dissipative. Analyses of this two-mode pressure-wave propagation have followed Biot (1956) and have examined a variety of limiting cases, based on the assumption of infinitesimal solid and fluid displacements in a fixed frame of reference (e.g., Garg *et al.*, 1974; Chandler and Johnson, 1981; Johnson and Plona, 1982). In the system I consider here, displacements are macroscopic rather than infinitesimal, and the frame of reference moves with the rod array. Consequently, to minimize mathematical manipulation, I use an *ad hoc* rather than formal approach.

The basic postulate of the *ad hoc* approach is that pore-pressure fluctuations due to elastic waves are inconsequential compared to pressure fluctuations associated with fluid flow and pore-pressure diffusion. Although Biot's (1956) theory shows that slow, diffusive ("type II") modes attenuate much more strongly than do fast, elastic ("type I") modes in instances of small deformation, I postulate that diffusion is more important in the sliding-array problem because it is associated with sustained fluid pressure imbalances that couple with the macroscopic motion of the rod array. The small speed and great attenuation of the diffusive mode are, in fact, precisely the properties that allow pressure imbalances to be sustained. (Infinitely fast modes with zero attenuation, as would occur in a perfectly rigid medium, would permit no pressure imbalances.) A corollary of this postulate is that, in the moving coordinate system, inertial effects in the pore fluid can be ignored. The deductive derivation addresses this simplification in terms of the Biot (1956) theory and shows that it is rigorously justified only if the value of a dimensionless parameter $B = (\mu n^2 \delta / k \rho_a v_x)$ is much greater than 1 and if $\rho_s(1 - n) >> \rho_f(n)$. (Here k is the hydraulic permeability of the solid porous medium and ρ_a is the dynamic added-mass density of both phases).

A second postulate, which extends the first, is that propagation of the slow, diffusive mode can be represented adequately with a zero-frequency approximation. In the limit of zero-frequency boundary forcing, the diffusive mode is governed by a simple, homogeneous diffusion equation, in which the dependent variable is the non-equilibrium pore pressure (Chandler and Johnson, 1981). The deductive formulation shows that the zero-frequency approximation is apt to be quite good for frequencies less than about 1000 Hz for cases in which the mass densities, porosities, and compressibilities of the solid and fluid constituents are similar to those of water-saturated soil or fragmented rock. Here I simply adopt the appropriate pore-pressure diffusion equation, although its validity rests upon the analysis of the deductive formulation and, ultimately, upon experimental test.

In the moving coordinate system the pore-pressure diffusion equation is

$$\frac{\partial p}{\partial t} = \frac{k}{\mu}\left(K_b + \frac{4}{3}G\right)\frac{\partial^2 p}{\partial y_m^2} \tag{9}$$

in which K_b and G are the drained elastic bulk modulus and shear modulus of the porous medium. Thus I assume that elastic strain of the porous medium influences the bulk (Darcian) motion of the fluid but not the bulk (translational) motion of the solid. Except for the use of a moving coordinate system, (9) is identical to the "weak-frame approximation" obtained by Chandler and Johnson (1981) as a special case of the Biot (1956) theory.

Initial and boundary conditions to be used in solving (9) must account for the flux of fluid at the slip surface as well as within the rod array. When the system is at rest, the fluid pressure is hydrostatic, so the initial condition specifies zero nonequilibrium pressure:

$$p(y_m, 0) = 0 \tag{10}$$

The boundary condition at the top of the moving array of rods, far from the slip surface, specifies that the fluid pressure remains hydrostatic:

$$p\left(\frac{\sqrt{3}}{2}M\delta, t\right) = 0 \tag{11}$$

The boundary condition along the slip surface derives from conservation of fluid and solid mass, which must be satisfied as the array of rods displaces fluid (cf. Deresiewicz and Skalak, 1963; Berryman and Thigpen, 1985). Consider the fluid volume, θ, in a single deforming pore along the slip surface (Fig. 1). If Q is the mass flux of fluid out of the top of the pore and into the porous medium, it must obey

$$-\frac{1}{\rho_f}Q = \frac{1}{2}\frac{d\theta}{dt} \tag{12}$$

The 1/2 appears in this equation because I assume that half the fluid efflux from the pore will be upward and half will be downward, and Q accounts for only the upward flux. It is convenient to divide Q by the cross-sectional area of the pore, yielding the quantity $q = Q/(\delta L)$ and the equation

$$-\frac{q}{\rho_f} = \frac{1}{2\delta L}\frac{d\theta}{dt} \tag{13}$$

Next I postulate that fluid flow out of the deforming pore and into the adjacent porous medium obeys Darcy's law, giving

$$q = -\rho_f \frac{k}{\mu}\frac{\partial p}{\partial y_m} \tag{14}$$

Note that Darcy's law must be written with reference to the moving coordinate y_m. This is consistent with the diffusion Eq. (9) and with the theory of Biot (1956). Combining (13) and (14) gives

$$\frac{\partial p}{\partial y_m} = \frac{\mu}{k}\frac{1}{2\delta L}\frac{d\theta}{dt} \tag{15}$$

This is a boundary condition on p at the base of the moving array of rods, but its utility is limited because it is expressed in terms of θ. From the geometry of Fig. 1 it is easy to deduce the relationship between u_y and θ:

$$\theta = L\left(\delta u_y - \frac{\pi}{4}\delta^2\right) \tag{16}$$

which leads to

$$\frac{d\theta}{dt} = \delta L\frac{du_y}{dt} \tag{17}$$

Substituting (17) into (15) then yields

$$\frac{\partial p}{\partial y_m}(0, t) = \frac{\mu}{2k}\frac{du_y}{dt} \tag{18}$$

which is the desired form of the boundary condition at the base of the moving rod array, where $y_m = 0$.

Thus, the mathematical model consists of two differential Eqs. (5) and (9) with the two unknowns p and u_y, as well as the five initial and boundary conditions (6), (7), (10), (11), and (18). Note that (5) is coupled to (9) via the non-equilibrium pressure-force term, whereas (9) is coupled to (5) via the boundary condition (18) and the definition of the moving coordinate y_m given by (8). Consequently, the system of equations must be solved simultaneously, as will be detailed in a subsequent paper.

DEDUCTIVE DERIVATION

Biot's (1956) dynamic mixture theory provides a framework for assessing the conditions under which the *ad hoc* mathematical model is valid. Biot focused on wave propagation in fluid-saturated, linearly elastic porous media, but he began his analysis by formulating general equations of motion that require neither linearly elastic behavior nor infinitesimal displacements. These equations can be specialized for one-dimensional motion without loss of physical generality, as long as the porous medium is isotropic. The one-dimensional equations for motion in the y direction are (Biot, 1956, Eq. 6.5)

$$\frac{\partial^2}{\partial t^2}(\rho_{11}u_y - \rho_a U_y) + \frac{\mu n^2}{k}\frac{\partial}{\partial t}(u_y - U_y) = f_y \tag{19a}$$

$$-\frac{\partial^2}{\partial t^2}(\rho_a u_y - \rho_{22}U_y) - \frac{\mu n^2}{k}\frac{\partial}{\partial t}(u_y - U_y) = F_y \tag{19b}$$

where u_y is the solid displacement and U_y is the fluid displacement; ρ_a, ρ_{11}, ρ_{22} are mass-density coefficients, and f_y and F_y represent all surface and body forces acting on the solid and fluid phases, respectively, per unit volume of the mixture.

Equations (19a) and (19b) reflect two types of coupling between the motion of the porous solid and pore fluid. The second term on the left-hand side of both (19a) and (19b) represents coupling due to the viscous drag force caused by the velocity difference between the two phases, $\partial(u_y - U_y)/\partial t$. This is the only dissipative force in Biot's formulation, and its form derives directly from his assumption that fluid flow is Darcian. The first, or acceleration, terms on the left-hand side of these equations represent the second type of coupling. This "inertia coupling" is not straightforward, as indicated by the presence of the phenomenological mass-density coefficients, ρ_{11}, ρ_{22}, ρ_a. Biot defined the coefficients as $\rho_{11} = \rho_1 + \rho_a$ and $\rho_{22} = \rho_2 + \rho_a$, where ρ_1 is the mass of the solid phase per unit volume of mixture ($\rho_1 = \rho_s - n\rho_s$) and ρ_2 is the mass of the fluid phase per unit volume of the mixture ($\rho_2 = n\rho_f$). The coefficient ρ_a is an apparent added-mass density of each phase due to the relative acceleration of the two phases. It accounts, for example, for the fact that the porous solid effectively has more inertia if it contains a finite mass of pore fluid. Thus coupled solid-fluid motion occurs even if the fluid is inviscid and there is no energy dissipation when one phase accelerates relative to the other.

Values of the added-mass density ρ_a are somewhat enigmatic, but are not impossible to estimate. For an isolated solid sphere accelerating in an ideal fluid, Batchelor (1967, p. 453 ff.) has calculated that $\rho_a = (1/2)\rho_f$, and Berryman (1980) has used this result to deduce, for a porous medium composed of a packed array of spheres, that $\rho_a = [(1 - n)/2]\rho_f$. In view of the analyses of Batchelor and Berryman, it is evident that the value of ρ_a is smaller than that of ρ_f but of the same order of magnitude. For example, using Batchelor's (1967, pp. 403–407) result for the added-mass density of a cylindrical rod accelerating in an ideal fluid, $\rho_a = \rho_f$, and applying Berryman's (1980) formula to a packed array of cylindrical rods, leads to the result that $\rho_a = (1 - n)\rho_f$ for the geometry of the porous medium of Fig. 2. Consequently, I conclude that the value of ρ_a

is somewhat smaller than but of the order of 1000 kg/m^3 both in the rod-array experiments of Iverson and LaHusen (1989) and in problems of water interaction with porous earth materials. Computational experiments by Yavari and Bedford (1990) support the generality of this conclusion.

I assess the relative importance of the dynamic terms in (19a) and (19b) by recasting and normalizing the equations. Introducing the relative displacement of the fluid with respect to the solid, $U_r = U_y - u_y$, and noting Biot's definitions of ρ_{11} and ρ_{22}, I rewrite (19a) and (19b) as

$$\frac{\partial^2}{\partial t^2}(\rho_1 u_y - \rho_a U_r) - \frac{\mu n^2}{k}\frac{\partial}{\partial t}(U_r) = f_y \tag{20a}$$

$$\frac{\partial^2}{\partial t^2}(\rho_2 U_y + \rho_a U_r) + \frac{\mu n^2}{k}(U_r) = F_y \tag{20b}$$

I then normalize the displacements with respect to the characteristic length δ, the time with respect to the characteristic time δ/v_x, the densities with respect to the added-mass density ρ_a, and the applied forces per unit volume with respect to $\rho_a g$. I accomplish this by multiplying each term in (20a) and (20b) by $\delta/v_x^2\rho_a$, which yields

$$\frac{\partial^2}{\partial t^{*2}}\left[\frac{\rho_1}{\rho_a}u_y^* - U_r^*\right] - B\frac{\partial U_r^*}{\partial t^*} = \frac{g\delta}{v_x^2}f_y^* \tag{21a}$$

$$\frac{\partial^2}{\partial t^{*2}}\left[\frac{\rho_2}{\rho_a}U_y^* + U_r^*\right] + B\frac{\partial U_r^*}{\partial t^*} = \frac{g\delta}{v_x^2}F_y^* \tag{21b}$$

in which

$$B = \frac{\mu n^2\delta}{k\rho_a v_x} \tag{22}$$

In these equations the asterisks denote normalized quantities.

The value of the dimensionless number B determines the relative importance of the viscous and inertial coupling terms involving the first and second time derivatives of U_r^* in (21a) and (21b). Values of B depend on the sliding velocity v_x (or characteristic forcing frequency v_x/δ) and on material properties. For some representative geological materials and sliding velocities and for the conditions of the experiments of Iverson and LaHusen (1989), Table 1 lists typical values of the parameters included in B as well as the values of B that result. The tabulated values show that for many materials and sliding rates, and certainly for the conditions of the rod-array experiments of Iverson and LaHusen (1989), $B >> 1$, indicating that viscous coupling dominates inertial coupling. Consequently I neglect the term $\partial^2 U_r^*/\partial t^{*2}$ in (21a and b), with the recognition that this disregards added-mass effects that may be important at high sliding

Table 1. Physical Properties and Model Parameters for Typical Water-Saturated Granular Media and Sliding Rates

Material[c]	δ(m)	ρ_s(kg/m^3)	k(m^2)	E_{solid}(Pa)	E_{bulk}(Pa)	ν_{bulk}	$K_b + \frac{4}{3}G$(Pa)	n	ρ_a(kg/m^3)	v_x(m/s)	B
Fiberglass rod array[a,b]	0.019	2300	1.8×10^{-11}	1.3×10^{9}	5.1×10^{6}	0.3	6.9×10^{6}	0.1	900	0.1–1	120–12
Fine gravel	0.01	2700	1×10^{-10}	1×10^{10}	10^{7}–10^{8}	0.33	10^{7}–10^{8}	0.3	350	0.1–10	300–3
Fine sand	0.0001	2700	1×10^{-12}	1×10^{10}	10^{7}–10^{8}	0.33	10^{7}–10^{8}	0.3	350	0.1–10	300–3
Cobble-sized fragmented rock	0.1	2700	1×10^{-8}	1×10^{10}	10^{7}–10^{8}	0.33	10^{7}–10^{8}	0.3	350	0.1–10	30–0.3
Poorly sorted landslide rubble	0.01	2700	1×10^{-11}	1×10^{10}	10^{7}–10^{8}	0.33	10^{7}–10^{8}	0.3	350	0.1–10	3000–30
Granite grus soil from Mount Tsukuba, Japan[b]	0.001	2630	5×10^{-11}	1×10^{10}	6×10^{6}	0.33	5×10^{6}	0.45	300	0.1–10	150–1.5
Fractured sandstone	0.001	2700	1×10^{-14}	1×10^{10}	10^{8}–10^{9}	0.3	10^{8}–10^{9}	0.1	600	0.1–10	17000–170
Intact igneous rock	0.001	2700	1×10^{-18}	1×10^{10}	10^{10}	0.25	10^{10}	<0.1	900	0.1–10	$>10^{6}$

[a] Data from Davis (1986).
[b] Data from Iverson and LaHusen (1989).
[c] For all materials it is assumed that the water properties are $\rho_f = 1000$ kg/m^3, $\mu = 10^{-3}$ Pa-s, $E = 2.3 \times 10^9$ Pa and that the gravitational acceleration is $g = 9.8$ m/s^2.

rates and forcing frequencies (above about 1000 Hz in the rod-array experiments).

The relative importance of the acceleration terms that involve u_y^* and U_y^* in (21a) and (21b) is more difficult to assess, because they do not involve the same dependent variable. However, conservation of mass dictates that u_y^*, U_y^*, and U_r^* typically are of the same order of magnitude, because solid displacements must be accompanied by compensating fluid displacements. The values of the dimensionless coefficients ρ_1/ρ_a and ρ_2/ρ_a that precede u_y^* and U_y^* are not typically of the same order of magnitude, however. Table 2 shows that the value of ρ_1/ρ_a typically exceeds that of ρ_2/ρ_a by about an order of magnitude. On these grounds I neglect the acceleration term $(\partial^2/\partial t^{*2})$ (ρ_2/ρ_a) U_y^*, so that the equation of motion for the fluid phase (21b) reduces to a quasistatic form. Consequently, the only acceleration term retained in (21a) and (21b) is that for the solid phase. This simplification is justified by scaling relationships, but not strongly so. Thus, it constitutes an hypothesis that must be tested against experimental data.

Fluid Motion and Pore-Pressure Diffusion

Adopting the simplifications described above and reverting to dimensional quantities [i.e., multiplying each term in (21b) by $(\rho_a v_x^2/\delta)$], Eq. (21b) assumes the quasistatic form

$$\frac{\mu n^2}{k}\frac{\partial U_r}{\partial t} = F_y = -n\left(\frac{\partial p_t}{\partial y} + \rho_f g\right) = -n\frac{\partial p}{\partial y} \tag{23}$$

The motive force on the fluid per unit volume of mixture F_y is simply equal to $-n(\partial p_t/\partial y + \rho_f g)$ or to $-n(\partial p/\partial y)$, in which p_t is the total pore pressure and p is the nonhydrostatic component of the pore pressure (cf. Biot, 1956). The fluid velocity relative to the solid is related to the fluid volumetric specific discharge relative to the solid q_r by (Bear, 1972, p. 209)

Table 2. Calculation of Normalized Mass-Density Coefficients for Some Typical Materials[a]

Material	n	ρ_s(kg/m^3)	ρ_f(kg/m^3)	ρ_a(kg/m^3)	ρ_1/ρ_a	ρ_2/ρ_a
Intact rock	0.1	2700	1000	450[b]	5.4	0.22
Fragmented rock, sand, gravel	0.3	2700	1000	350[b]	5.4	0.85
Fiberglass rod array	0.1	2300	1000	900[c]	2.3	0.11

[a] By definition, $\rho_1 = \rho_s(1 - n)$; $\rho_2 = \rho_f(n)$.
[b] Calculated using Berryman's (1980) formula for an assembly of spheres: $\rho_a = [(1 - n)/2]\,\rho_f$.
[c] Calculated $\rho_a = (1 - n)\rho_f$ for an assembly of cylindrical rods.

$$\frac{\partial U_r}{\partial t} = \frac{q_r}{n} \tag{24}$$

and substitution of (24) into (23) yields the conventional form of Darcy's law

$$q_r = -\frac{k}{\mu}\frac{\partial p}{\partial y} \tag{25}$$

If a linearly elastic solid rheology is assumed, Darcy's law can be combined with mass-conservation and poroelastic constitutive equations to obtain various coupled and uncoupled diffusion equations, which describe the quasistatic distributions of pore pressure and solid stress that result from transient pore-pressure fluctuations (Biot, 1941; Rice and Cleary, 1976; Roeloffs, 1988). Such diffusion equations represent a special case of Biot's (1956) dynamic theory (Chandler and Johnson, 1981; Johnson and Plona, 1982). The extent of solid-fluid coupling in these equations depends on loading conditions, domain geometry, and the relative compressibilities of the individual phases and the bulk mixture. Here I simplify the coupling problem by assuming that the bulk composite is much more compressible than are the solid and fluid constituents. Table 1 lists elastic moduli (i.e., reciprocal compressibilities) for the fiberglass rod array of Iverson and LaHusen (1989) and for typical geologic media. The tabulated data show that the incompressible-constituent assumption is appropriate for modeling the rod-array experiments and for geologic materials in which the solid phase is fractured or disaggregated into soil grains or rock fragments.

Employing the incompressible-constituent assumption and restricting attention to one-dimensional motion of both phases, conservation of mass for the fluid phase yields (cf. Bear, 1972, p. 205)

$$\frac{\partial q_r}{\partial y} + \frac{\partial}{\partial y}(n v_y) + \frac{\partial n}{\partial t} = 0 \tag{26}$$

in which v_y is the y component of the solid phase velocity. In this equation the second term results from the fact that q_r is evaluated with respect to the moving solids, so that $q_r = q_y - n v_y$, where q_y is the specific discharge with respect to fixed, Eulerian coordinates. Concurrently, conservation of mass for the solid phase is described by

$$\frac{\partial}{\partial y}[(1 - n)v_y] + \frac{\partial(1 - n)}{\partial t} = 0 \tag{27}$$

in which the argument of the space derivative can be thought of as the specific discharge of the solid phase (Bear, 1972, p. 208). Addition of (26) and (27) yields a simple balance between the one-dimensional divergences of the fluid specific discharge and solid-phase velocity

$$\frac{\partial q_r}{\partial y} = -\frac{\partial v_y}{\partial y} \tag{28}$$

A second equation for the divergence of the fluid specific discharge results from differentiation of Darcy's law (25)

$$\frac{\partial q_r}{\partial y} = -\frac{k}{\mu}\frac{\partial^2 p}{\partial y^2} \tag{29}$$

and combination of (28) and (29) yields

$$\frac{\partial v_y}{\partial y} = \frac{k}{\mu}\frac{\partial^2 p}{\partial y^2} \tag{30}$$

To express (30) in terms of p alone requires a relationship between the solid-phase velocity divergence and non-equilibrium pore pressure. This can be obtained by considering the constitutive relationship between effective stress and elastic strain, which is assumed to be small and governed by Hooke's law. The Hookean (linear elastic) constitutive relationship for a fluid-saturated, isotropic porous medium is (cf. Rice and Cleary, 1976; Iverson and Reid, 1992)

$$\epsilon_{ij} = \frac{1+\nu}{E}\sigma'_{ij} - \frac{\nu}{E}\sigma'_{kk}\delta_{ij} \tag{31}$$

where ϵ_{ij} denotes a component of the Eulerian small-strain tensor, σ'_{ij} denotes a component of effective stress, E is Young's modulus, ν is Poisson's ratio, and δ_{ij} is the Kronecker delta. To obtain an equation specialized for one-dimensional extension or compression confined to the y direction, I consider the case in which the only nonvanishing component of strain is ϵ_{yy} and in which $\sigma'_{xx} = \sigma'_{zz} \neq 0$ are reaction stresses that result from an imposed stress σ'_{yy}. In this case I obtain from Hooke's law (31)

$$\epsilon_{yy} = \frac{1-2\nu}{E}\left(\sigma'_{yy} + \frac{2\nu}{1-\nu}\sigma'_{yy}\right) \tag{32}$$

Employing standard relationships between elastic moduli (Fung, 1965, pp. 129–130), (32) reduces to a simple expression relating the longitudinal strain to the imposed stress in terms of the elastic bulk modulus K_b and shear modulus G

$$\sigma'_{yy} = (K_b + \tfrac{4}{3}G)\epsilon_{yy} \tag{33}$$

If the solid and fluid constituents of the porous medium are effectively incompressible, as I assume here, then the standard definition of effective stress in a fluid-saturated elastic medium (Nur and Byerlee, 1971) reduces to

$$\sigma'_{ij} = \sigma_{ij} + p_t\delta_{ij} = \sigma_{ij} + (p - \rho_f g y)\delta_{ij} \tag{34}$$

in which σ_{ij} is the total stress. This equation shows that the total pore pressure, including the hydrostatic pressure, influences the effective stress if body forces are included in the formulation. For the one-dimensional case I consider here, (34) reduces to

$$\sigma'_{yy} = \sigma_{yy} + p - \rho_f g y \tag{35}$$

To obtain the desired relationship between the solid velocity divergence and p, I first consider the material time derivative of the effective stress (35),

$$\frac{D\sigma'_{yy}}{Dt} = \frac{\partial p}{\partial t} + v_y \frac{\partial p}{\partial y} + \frac{\partial \sigma_{yy}}{\partial t} + v_y \frac{\partial \sigma_{yy}}{\partial y} - v_y \rho_f g \tag{36}$$

For the sliding rod-array problem, in which the only time-dependent loading is due to changes in pore pressure and elevation of the solids, the last three terms on the right-hand side of (36) sum to zero. This can be recognized by considering a case in which the solid translates vertically through the fluid without generating a non-equilibrium pore pressure (e.g., a case in which the fluid has zero viscosity or the porous medium has infinite permeability). Then $p = 0$ everywhere, but changes in total stress σ_{yy} occur because changes in static fluid pressure on the solid result from the solid translation. Concurrently, translation does not influence the effective stress ($D\sigma'_{yy}/Dt = 0$), because changes in total stress and hydrostatic pore pressure cancel one another. Thus the last three terms in (36) sum to zero—a result unaffected by the presence of nonzero p, as long as there is no other time-dependent loading. Thus (36) reduces to

$$\frac{D\sigma'_{yy}}{Dt} = \frac{\partial p}{\partial t} + v_y \frac{\partial p}{\partial y} \tag{37}$$

Substitution of (33) into (37) results in

$$\frac{D\epsilon_{yy}}{Dt} = \frac{1}{K_b + \frac{4}{3}G}\left(\frac{\partial p}{\partial t} + v_y \frac{\partial p}{\partial y}\right) \tag{38}$$

The material time derivative of the Eulerian strain in (38) equals the divergence of the solid velocity v_y (Mase, 1970, p. 113)

$$\frac{D\epsilon_{yy}}{Dt} = \frac{\partial \epsilon_{yy}}{\partial t} + v_y \frac{\partial \epsilon_{yy}}{\partial y} = \frac{\partial v_y}{\partial y} \tag{39}$$

Combination of (39), (38), and (30) results in an equation for p alone

$$\frac{\partial p}{\partial t} + v_y \frac{\partial p}{\partial y} - \frac{k}{\mu}\left(k_b + \frac{4}{3}G\right)\frac{\partial^2 p}{\partial y^2} = 0 \tag{40}$$

Equation (40) is an advective-diffusion equation for the non-equilibrium pore pressure, in which $k/\mu\ [K_b + (4/3)G]$ plays the role of a pore-pressure diffusivity. Except for inclusion of the advection term, $v_y(\partial p/\partial y)$, (40) is identical to the pore-pressure diffusion equation obtained by Chandler and Johnson (1981), who assessed the zero-frequency limit of the Biot (1956) wave-propagation theory. Chandler and Johnson (1981) obtained their equation by considering the appropriate reduction of the Biot theory for cases in which the solid and fluid constituents are effectively incompressible, as I have assumed here. They also incorporated Biot's assumption that the solid displacements as well as strains are small, that is, that rigid-body translation is inconsequential. If translation is significant, a pore-pressure advection term such as that in (40) is required.

In a coordinate system that translates with velocity v_y, (40) reduces to a standard diffusion equation (cf. Ogata, 1970). Thus I employ the moving coordinate y_m defined in (8) and note that $v_y = \partial u_y/\partial t = \partial(y - y_m)/\partial t = -\partial y_m/\partial t$ to obtain from (40):

$$\frac{\partial p}{\partial t} - \frac{k}{\mu}\left(K_b + \frac{4}{3}G\right)\frac{\partial^2 p}{\partial y_m^2} = 0 \tag{41}$$

This matches the diffusion equation adopted in the inductive derivation.

Solid Motion

The scaling relationships used to obtain (41) indicate that the largest acceleration term in the coupled equations of motion (21a and b) is that for the solid phase, $\partial^2/\partial t^{*2}\ [(\rho_1/\rho_a)\ u_y^*]$. Thus, retaining this term in (21a), assuming that $B >> 1$, and reverting to dimensional quantities, the solid equation of motion becomes

$$\frac{\partial^2}{\partial t^2}(\rho_1 u_y) - \frac{\mu n^2}{k}\frac{\partial}{\partial t}(U_r) = f_y = (1 - n)\left(\frac{\partial \sigma_{yy}}{\partial y} - \rho_s g\right) \tag{42}$$

The force on the solid per unit volume of mixture, f_y, is given by the product of the solid volume fraction $1 - n$ with the total stress gradient ($\partial\sigma_{yy}/\partial y$ is the only nonvanishing component of the total stress gradient) and gravitational body force. Thus f_y is precisely analogous to F_y that acts on the fluid phase (cf. 23). Solving the effective-stress Eq. (35) for the total stress σ_{yy}, then differentiating with respect to y, and substituting the resulting expression into (42) yields

$$\frac{\partial^2}{\partial t^2}(\rho_1 u_y) - \frac{\mu n^2}{k}\frac{\partial}{\partial t}(U_r) = (1 - n)\left(\frac{\partial \sigma'_{yy}}{\partial y} - \rho_s g - \frac{\partial p}{\partial y} + \rho_f g\right) \tag{43}$$

I now assume that the longitudinal strain is an insignificant part of the solid motion, that is, that macroscopic rigid-body translation far exceeds displacements associated with internal strain. Thus I take $\epsilon_{yy} = 0$, which implies, by (33), that, $\sigma'_{yy} = 0$ in (43). I also employ (23) to substitute $-n(\partial p/\partial y)$ for the second term on the left-hand side of (43) and the definition $\rho_1 = (1 - n)\rho_s$ to reformulate the first term on the left-hand side. Making these substitutions in (43) and canceling the redundant term $n(\partial p/\partial y)$ from each side of the resulting equation yields

$$\frac{\partial^2}{\partial t^2}[(1 - n)\rho_s u_y] = -(1 - n)(\rho_s - \rho_f)g - \frac{\partial p}{\partial y} \tag{44}$$

Since I regarded the solid as effectively rigid in deriving (44), the partial time derivative can be replaced by a total time derivative, $(1 - n)\rho_s$ can be moved outside the differential, and the equation can be integrated easily over finite intervals of the space coordinates x, y, and z. Integrating (44) over the intervals $x = 0$ to $x = \delta$, $z = 0$ to $z = L$, and $y = u_y - \delta/2$ to $y = u_y - \delta/2 + (\sqrt{3}/2)M\delta$, and dividing each term by $(\sqrt{3}/2)\delta^2 ML(1 - n)\rho_s$ yields

$$\frac{d^2 u_y}{dt^2} = -\left(1 - \frac{\rho_f}{\rho_s}\right)g - \frac{2}{\sqrt{3}\,\delta M \rho_s (1 - n)} \cdot \left[p\left(u_y - \delta/2 + \frac{\sqrt{3}}{2}M\delta, t\right) - p(u_y - \delta/2, t)\right] \tag{45}$$

which matches (5) obtained in the inductive derivation.

Thus the differential equations that govern coupled solid-fluid motion, (41) and (45), deduced from the general theory of Biot (1956) by making specific, quantitative assumptions, match the corresponding equations obtained using an inductive approach. The initial and boundary conditions, (6), (7), (10), (11), and (18), complete the specification of the mathematical model.

SUMMARY AND CONCLUSIONS

A mathematical model of the origin and propagation of dynamic pore-pressure fluctuations in a rapidly sliding, water-saturated granular medium can be derived by either an inductive or deductive method. The inductive method constructs governing equations through an analysis of the mechanics of the sliding array of cylindrical fiberglass rods studied experimentally by Iverson and LaHusen (1989). The deductive method extracts the same equations mathematically from the mixture theory originally developed by Biot (1956) to investigate poroelastic wave propagation. The deductive derivation supplements the inductive derivation by quantifying conditions that must be satisfied to assure the validity of the governing equations.

The governing equations require that seven principal conditions be satisfied:

1. Horizontal motion of the sliding rod array is steady. Transient motion is exclusively vertical, in the y direction.

2. Internal deformation (strain) of the rod array is infinitesimal and linearly elastic, and ϵ_{yy} is the only nonzero strain component.

3. Vertical translation of the rod array greatly exceeds displacements due to internal strain of the array.

4. Fluid flow through the rod array is Darcian.

5. The rod array is much more compressible than are either the solid rods themselves or the pore fluid. Thus compression occurs almost exclusively by pore-space reduction.

6. The dimensionless number $B = (\mu n^2 \delta / k \rho_a v_x)$ is much larger than one, implying that solid-fluid coupling forces are dominated by viscous rather than inertial effects. This restriction allows neglect of added-mass effects on acceleration of the solid and fluid phases. It places an upper bound on the sliding velocity or forcing frequency for which the model is valid. In the case of the fiberglass rod experiments reported by Iverson and LaHusen (1989), and for many landslide problems, it limits the strict applicability of the model to sliding rates less than about 10 m/s.

7. Fluid-inertia effects are negligible compared to solid-inertia effects, so that the only acceleration term retained in the equations of motion is that for the solid array. This assumption is applicable if $\rho_1 \gg \rho_2$, where $\rho_1 = \rho_s(1 - n)$ and $\rho_2 = \rho_f(n)$. This condition is satisfied, but only marginally, in the fiberglass rod experiments and in many geophysical problems. Consequently, this condition is probably the most restrictive of any implicit in the mathematical model. Its effect on model predictions must be examined in light of experimental tests.

The basal boundary condition used to solve the pore-fluid equation of motion has been derived only for the specific conditions of the rod-array experiments of Iverson and LaHusen (1989). More general boundary conditions need to be developed for application to typical geophysical phenomena.

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APPENDIX: SYMBOL DEFINITIONS AND DIMENSIONS

B: dimensionless number obtained from normalization of Biot's equations.

E: Young's modulus of elasticity $[M/LT^2]$.

F: Force acting on solids in unit cell $[ML/T^2]$.

F_g: Gravity force acting on solids in unit cell $[ML/T^2]$.

F_p: Nonequilibrium pore-pressure force acting on solids in unit cell $[ML/T^2]$.

f_y: Force per unit volume of mixture acting on solid phase $[M/L^2T^2]$.

F_y: Force per unit volume of mixture acting on fluid phase $[M/L^2T^2]$.

g: Magnitude of gravitational acceleration $[L/T^2]$.

G: Shear modulus of elasticity $[M/LT^2]$.

i, j: Dummy indices (subscripts) that represent Cartesian coordinate directions $[L]$.

k: Hydraulic permeability of porous medium $[L^2]$.

K_b: Bulk modulus of elasticity $[M/LT^2]$.

L: Length of unit cell of rod array $[L]$.

m: Mass of solids in unit cell $[M]$.

M: Number of rods stacked atop the slip surface in unit cell (dimensionless).

n: Porosity (dimensionless).

p: Nonequilibrium pore pressure $[M/LT^2]$.

p_t: Total pore pressure $[M/LT^2]$.

q: Mass efflux of fluid out of a pore at the slip surface, per unit area $[M/L^2T]$.

q_r: Volumetric fluid specific discharge relative to the solid $[L/T]$.

Q: Mass efflux of fluid out of a pore at the slip surface $[M/T]$.

t: Time $[T]$.

u_y: Solid-phase displacement $[L]$.

U_y: Fluid-phase displacement $[L]$.

U_r: Relative displacement of solid and fluid phases $[L]$.

v_x: Horizontal component of solid velocity $[L/T]$.

v_y: Vertical component of solid velocity $[L/T]$.

x, y: Cartesian coordinates in horizontal and vertical directions $[L]$.

y_m: Vertical coordinate that translates with the moving solid array $[L]$.

δ: Rod diameter $[L]$.

δ_{ij}: Kronecker delta ($= 1$ when $i = j$; $= 0$ when $i \neq j$) (dimensionless).

ϵ_{ij}: Component of elastic strain (dimensionless).

μ: Fluid viscosity $[M/LT]$.

ν: Poisson's ratio of elasticity (dimensionless).

ρ_a: Added-mass density due to inertia coupling of solid and fluid motion $[M/L^3]$.

ρ_f: Fluid mass density $[M/L^3]$.

ρ_s: Solid mass density $[M/L^3]$.
ρ_1: Mass of solid per unit volume of mixture $[M/L^3]$.
ρ_2: Mass of fluid per unit volume of mixture $[M/L^3]$.
ρ_{11}: Defined as $\rho_1 + \rho_a$ $[M/L^3]$.
ρ_{22}: Defined as $\rho_2 + \rho_a$ $[M/L^3]$.
σ_{ij}: Total stress $[M/LT^2]$.
σ'_{ij}: Effective stress $[M/LT^2]$.
θ: Fluid volume in a pore along the slip surface $[L^3]$.
*: Superscript denotes normalized (dimensionless) quantity.

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