New views of granular mass flows

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ABSTRACT

Concentrated grain-fluid mixtures in rock avalanches, debris flows, and pyroclastic flows do not behave as simple materials with fixed rheologies. Instead, rheology evolves as mixture agitation, grain concentration, and fluid-pressure change during flow initiation, transit, and deposition. Throughout a flow, however, normal forces on planes parallel to the free upper surface approximately balance the weight of the superincumbent mixture, and the Coulomb friction rule describes bulk intergranular shear stresses on such planes. Pore-fluid pressure can temporarily or locally enhance mixture mobility by reducing Coulomb friction and transferring shear stress to the fluid phase. Initial conditions, boundary conditions, and grain comminution and sorting can influence pore-fluid pressures and cause variations in flow dynamics and deposits.

Keywords: landslide, avalanche, debris flow, pyroclastic flow, mass movement, mechanics.

INTRODUCTION

Granular mass flows include rock avalanches, debris flows, pyroclastic flows, and other phenomena in which gravity drives rapid downslope motion of grains and intergranular fluid. The word "granular" highlights the importance of momentum transport by large (>0.06 mm) solid grains, mixed with less-dense intergranular liquid or gas. The word "mass" implies that a finite, contiguous body of solid and fluid moves almost in unison, and the word "flow" indicates that the grain-fluid body deforms irreversibly as it moves downslope.

High volumetric grain concentrations distinguish granular mass flows from phenomena such as density currents and water floods, in which fluid forces dominate momentum transport and weak interactions of suspended solids influence effective fluid properties (cf. Bagnold, 1956; Lowe, 1976; Middleton, 1993). Effects of volumetric grain concentration (υ_s) on momentum transport can be quantified in a rudimentary way by evaluating the ratio of solid mass to fluid mass per unit volume,

$$N_{\text{mass}} = \frac{\rho_s \upsilon_s}{\rho_f(1 - \upsilon_s)} = \frac{\rho_s \upsilon_s}{\rho - \rho_s \upsilon_s}, \qquad (1)$$

in which ρ_s and ρ_f are the mass densities of the solid grains and intergranular fluid, respectively, and $\rho = \rho_s\upsilon_s + \rho_f(1 - \upsilon_s)$ is the mass density of the mixture. Values of $N_{\text{mass}} > 1$ imply that momentum transport by solid grains may dominate (e.g., Table 1).

Many investigators have proposed that rheological equations relating shear stress and shear rate hold the key for understanding and predicting the behavior of granular mass flows (e.g., Johnson, 1970; Iverson, 1985; Pierson and Costa, 1987). Diverse rheological formulae have been used to model flow dynamics and interpret deposits, but independent constraints on assumed rheologies and stringent tests of model predictions typically have been lacking (e.g., Hsü, 1975; Wilson and Head, 1981; Chen, 1988; Sousa and Voight, 1991; Whipple and Dunne, 1992; Battaglia, 1993; Palladino and Valentine, 1995). In contrast, well-constrained laboratory experiments indicate that even a highly simplified granular mass flow can exhibit variations in apparent rheology (Fig. 1). In this paper we summarize reasons for eschewing the fixed-rheology paradigm, and we describe a new view of granular mass flows that is consistent with observations and data.

RHEOLOGY AND CHANGES OF STATE

Fixed rheological relations between stress and shear rate apply only to simple materials that undergo no changes of state. For example, Newton's law of viscosity can adequately describe the rheology of liquid water, but not that of solid ice. The assumption that rheological laws exist for grain-fluid mixtures perhaps derives from Einstein's (1906) success in deducing the effective viscosity of dilute mixtures of small solid spheres suspended in Newtonian liquids. However, numerous mea-

TABLE 1. VALUES OF KEY DIMENSIONAL AND DIMENSIONLESS PARAMETERS ESTIMATED FOR UNFLUIDIZED PARTS OF SOME WELL-DOCUMENTED GRANULAR MASS FLOWS

Parameter	Flow location and type (with data source)				
	Miniature flume debris flow (Figure 1)	USGS flume debris flow (Iverson, 1997)	Kamikamihorizawa debris flow (Takahashi, 1991)	Mount St. Helens pyroclastic flows (Wilson and Head, 1981; Kuntz et al., 1981; Hoblitt, 1986)	Elm rock avalanche (Hsü, 1975, 1978)
h (m)	0.04	0.2	2	1	5
ρ_s (kg/m^3)	2700	2700	2700	2600	2400
ρ_f (kg/m^3)	1000	1000	1000	2	2
υ_s (none)	0.5	0.6	0.6	0.3	0.5
δ (m)	0.006	0.01	0.2	0.02	0.5
γ (s^{-1})	30	50	3	10	5
μ (Pa-s)	0.001	0.01	0.1	2×10^{-5}	2×10^{-5}
N_{mass} (none)	3	4	4	600	1000
N_{Sav} (none)	0.1	0.2	0.03	0.004	0.1
N_{Bag} (none)	8000	6000	10 000	9×10^5	4×10^8

Note: Definitions and methods of estimation of physical parameters:
h typical flow depth, estimated from observations and measurements of flow and deposit thicknesses
ρ_s solid grain mass density estimated from densities of grain in deposits
ρ_f pore fluid mass density estimated from typical densities of liquid water or dusty air, as appropriate
υ_s solid volume fraction, estimated from real-time measurements or deposit bulk densities
δ typical grain diameter, estimated from sampled deposits and descriptive accounts
γ typical shear rate, estimated by dividing observed flow speeds by typical flow thicknesses
μ pore-fluid viscosity, estimated from typical values for water, muddy water, and dusty air, as appropriate
Values of all parameters except ρ_s are rounded to one significant digit.

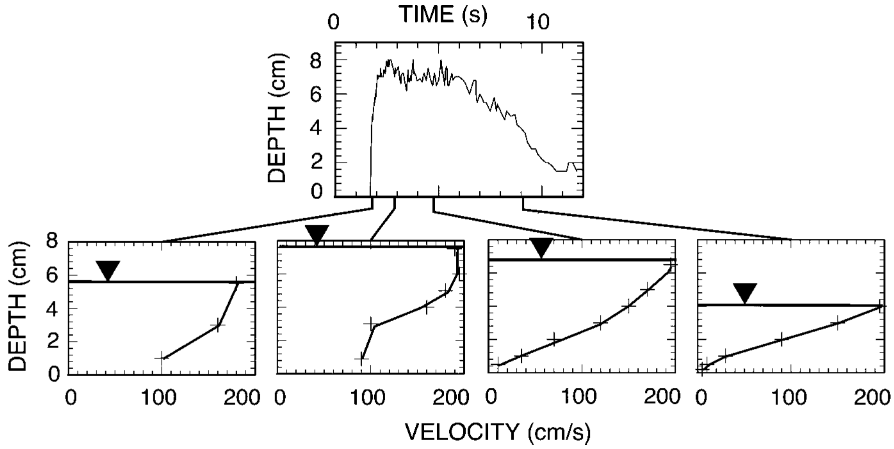

Figure 1. Variation of flow depths and vertical velocity profiles in small-scale experimental debris flow as it passed cross section 3 m downslope from head of rectangular flume 0.3 m wide and sloped 18°. Variations in velocity profiles reflect variations in apparent rheology. Flume bed was roughened with grains like those in flowing debris: sizes ranged from 0.8 mm to 20 mm; mean was 6 mm. Data were extracted from high-speed videotape recordings of flow past smooth glass flume walls. Volumetric grain concentrations υ_s determined by grab sampling varied from 0.5 at flow head, which was unsaturated, to 0.45 at flow tail, which was water saturated. Flow was initiated by pouring water at 0.008 m^3/s into static 0.1 m^3 heap of grains placed on flume bed.

surements and observations of concentrated grain-fluid mixtures (typically with $\upsilon_s > 0.4$) reveal their tendency to act almost like rigid-plastic solids if pore-fluid pressures and mixture agitation are negligible, but to flow almost like liquids when fluid pressures or agitation are sufficiently great (Jaeger et al., 1996; Duran, 2000). This contrasting behavior provides strong evidence that grain-fluid mixtures can exist in different states, somewhat analogous to conventional solid and liquid states. Values of variables that describe these states evolve as flow proceeds from initiation to deposition, and this evolution fundamentally distinguishes state variables from rheological constants.

Two state variables that conspicuously influence granular mass flows are nonequilibrium pore-fluid pressure, *p*, which is ambient fluid pressure minus the pressure due to fluid weight, and granular temperature, *T*, which is a scalar measure of grain vibration (i.e., mixture agitation) (Iverson, 1997). Both *p* and *T* are zero in static equilibrium states; both can increase during a mass flow through conversion of bulk potential and kinetic energy to internal mechanical energy, and both can decrease through degradation of mechanical energy to thermodynamic heat. Moreover, *p* and *T* can spread from one part of a flow to another by diffusive processes.

Increases in *T* decrease mixture rigidity (i.e., enhance fluidity) but increase energy dissipation due to inelastic grain collisions. Apparently rigid plugs can exist in flows where *T* is small enough for grains to maintain an interlocking structure (e.g., Hui and Haff, 1986; Drake, 1990). This view differs fundamentally from the rheological view that plug flow results from intrinsic yield strength independent of the mixture state.

Pore-fluid pressure in excess of static equilibrium pressure plays a role partly analogous to that of *T*, because it reduces mixture rigidity and facilitates granular flow. However, in contrast to high *T*, high *p* can reduce net energy dissipation by reducing grain-contact stresses and transferring shear stress to the fluid phase (Iverson, 1997).

Abundant field evidence illustrates the effects of variable granular temperature and pore-fluid pressure on granular mass flows and their deposits. For example, debris flows, rock avalanches, and pyroclastic flows commonly originate from nearly rigid sources, move fluidly while agitated and in transit, and form deposits that lock frictionally and eventually consolidate to a nearly rigid state. Freshly emplaced deposits of debris flows and pyroclastic flows typically have strong perimeters that enclose weak, almost fluidized interiors (e.g., Fig. 2). Grain-size sorting contributes to these contrasting states by promoting dissipation of nonequilibrium pore-fluid pressure in coarse-grained margins that develop during flow (Major and Iverson, 1999).

Figure 2. Photograph of freshly emplaced pyroclastic flow deposit ~6 km north of crater of Mount St. Helens, Washington, July 22, 1980. Deposit lobes had steep, high-strength margins of coarse clasts with typical diameters ~0.1 m. For weeks following deposition, finer-grained deposit interior (with typical grain diameters ~1 mm) remained warm, weak, and partially fluidized by escaping gas (Wilson and Head, 1981). Photo courtesy of R.P. Hoblitt.

BAGNOLD AND SAVAGE NUMBERS

Bagnold's (1954) famous shear-cell experiments have helped illuminate granular mass flows, but have commonly been misinterpreted in a rheological context. Bagnold's (1954) most consistent finding was that bulk intergranular shear stresses τ and normal stresses σ on planes of shear in mixtures of neutrally buoyant grains and liquid maintained a nearly constant proportion, $\tau/\sigma \approx \tan\psi_d$, where ψ_d is a dynamic friction angle independent of shear rate $\dot{\gamma}$. Subsequent shear-cell experiments with and without intergranular liquid largely confirmed this key finding and demonstrated that the dynamic friction angle typically differs only slightly from the static friction angle ψ_s (e.g., Savage and McKeown, 1983; Savage and Sayed, 1984; Hungr and Morgenstern, 1984a; Hanes and Inman, 1985).

Rheology advocates have instead emphasized that grain collisions in Bagnold's (1954) enclosed shear cell produced stresses (σ and τ) roughly proportional to either $\dot{\gamma}$ or γ^2. This proportionality depended on the value of N_{Bag}, a dimensionless number that estimates the ratio of grain-collision stresses to viscous shear stresses in a granular mixture with no gravitational stresses at grain contacts,

$$N_{\mathrm{Bag}} = \frac{\rho_s \dot{\gamma} \delta^2}{\mu} \lambda^{1/2}. \qquad (2)$$

Here μ is the viscosity of intergranular fluid, δ is the grain diameter, and λ is the linear

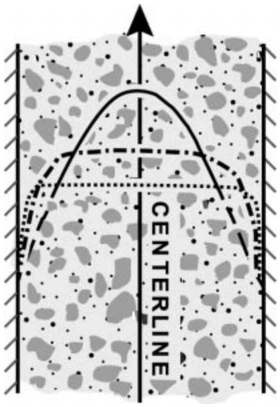

Figure 3. Schematic illustration of gravity-driven granular mass flow in channel bounded by vertical walls. Arrow denotes flow direction. Various velocity profiles transverse to flow direction are possible, as shown, but such profiles are inadequate to constrain flow rheology (cf. Johnson, 1970; Hui and Haff, 1986). Symmetry dictates that all profiles have zero shear rate ($\dot{\gamma}$) at channel centerline.

concentration of spherical grains, related to the volumetric concentration υ_s by $\lambda = \upsilon_s^{1/3}/(\upsilon_*^{1/3} - \upsilon_s^{1/3})$, where υ_* is the maximum possible grain concentration.

Contradictions arise when Bagnold's equations relating stresses to $\dot{\gamma}$ or γ^2 are used as rheological formulas, however. For example, consider points of zero shear rate ($\dot{\gamma} = 0$) that must be present in gravity-driven flows in channels (Fig. 3). Bagnold's equations predict that intergranular normal stress as well as shear stress must vanish at such points—a clear contradiction to the existence of gravitational driving stresses. Bagnold's (1954) shear-cell experiments lacked this contradiction because he intentionally camouflaged the effect of gravity on grain-contact stresses by using neutrally buoyant grains.

A more relevant interpretation of Bagnold's (1954) findings takes account of gravity stresses borne by dense grains surrounded by less-dense fluid. Savage (1984) introduced a dimensionless number that estimates the ratio of grain-collision stresses to gravitational stresses in steady, gravity-driven flows with free upper surfaces. Generalized to account for the effects of buoyancy due to pore-fluid weight, Savage's number can be written as

$$N_{Sav} = \frac{\rho_s \dot{\gamma}^2 \delta^2}{(\rho_s - \rho_f) g h}, \qquad (3)$$

where g is the magnitude of gravitational acceleration and h is depth below the flow surface. On the basis of diverse data, Savage and Hutter (1989) estimated that grain-collision stresses become important when $N_{Sav} > 0.1$. Otherwise, gravity dominates stresses at grain contacts, and N_{Bag} is largely irrelevant. Even rather rapid granular mass flows can have $N_{Sav} \leq 0.1$ at typical depths $h \gg \delta$, indicating that gravity dominates stresses at most grain contacts (Table 1).

COULOMB STRESSES

The prevalence of gravitationally induced grain-contact stresses, indicated by small values of N_{Sav}, leads to the hypothesis that Coulomb friction commonly generates most shear resistance in granular mass flows. We suggest that the Coulomb (1776) friction equation with zero cohesion,

$$\tau = \sigma \tan \varphi_s, \qquad (4)$$

is the best-tested and most parsimonious model for bulk stresses in granular mixtures that deform macroscopically (cf. Häner and Spencer, 1998; Duran, 2000). The Coulomb equation differs from rheological equations typically applied to granular mass flows because it implies no dependence of stress on shear rate. Instead, shear and normal stresses in a mass flow simply maintain Coulomb proportionality and satisfy momentum conservation (Iverson and Denlinger, 2001). Experiments with gravity-driven grain flows indicate that the Coulomb equation yields good predictions even when flow is rapid or partially liquefied by high fluid pressure (e.g., Hungr and Morgenstern, 1984b; Savage and Hutter, 1989; Denlinger and Iverson, 2001). This behavior may result from strong similarity of the Coulomb equation and the Bagnold equation $\tau = \sigma \tan \varphi_d$ for intergranular shear stress. The two equations predict essentially the same intergranular shear stress if normal stress on shear planes is the same, regardless of shear rate.

FREE SURFACE EFFECTS

The Coulomb proportionality between intergranular normal stresses and shear stresses indicates that free upper surfaces play a crucial role in granular mass flows. Intergranular normal stress σ on planes at depth h in a gravity-driven flow with a free upper surface depends on grain and fluid densities, grain concentration υ_s, the angle of surface inclination θ, and the nonequilibrium component of intergranular fluid pressure p, such that

$$\sigma = (\rho_s - \rho_f)\upsilon_s gh \cos\theta - p \qquad (5)$$

applies exactly if flow thickness is constant and otherwise applies approximately. Equation 5 is valid regardless of whether grains contact one another statically or dynamically collide. (As an illustration of this principle, consider the time-averaged normal stress imparted to a rigid bed by a freely bouncing ball; a simple analysis employing Newton's second and third laws of motion confirms that the stress is the same as if the ball were resting on the bed.)

Equation 5 indicates that intergranular normal stresses on planes at depth h depend linearly on υ_s. In turn, changes in υ_s accompany repositioning of the free upper surface that accompanies changes in mixture agitation and fluid pressurization. This interdependence of υ_s, T, p, and free-surface height fundamentally distinguishes granular mass flows from flows enclosed in shear cells. However, changes in υ_s necessary to balance changes in surface height are generally slight; experiments indicate that mean grain concentrations in gravity-driven granular mass flows with $N_{Sav} \leq 0.1$ generally remain close to values ($\upsilon_s \sim 0.4$–0.6) typical of loosely packed, static sediment heaps (Fig. 1; cf. Hungr and Morgenstern, 1984b; Iverson, 1997). Consequently, intergranular shear stresses can be readily estimated with simple equations such as 4 and 5.

UNSTEADY FLOW AND DEPOSITION

Granular mass flows have definite starting and ending points in space and time, and their motion is clearly unsteady. Abrupt surge fronts form at the heads of most flows, followed by thinner, tapering tails (e.g., Fig. 1). Changes in T, υ_s, and free-surface height during unsteady motion allow nonequilibrium pore-fluid pressure ($p \neq 0$) to exist (Iverson, 1997), and a single value of N_{Sav} is therefore unlikely to apply throughout the extent and duration of a flow. Pore-fluid pressures greater than static equilibrium pressures can transiently or locally increase the effective value of N_{Sav} by reducing gravitational stresses at grain contacts and mimicking the condition $\rho_f \rightarrow \rho_s$. In the limit of complete mixture liquefaction by high p, gravitational stresses at grain contacts vanish, and N_{Bag} describes stress partitioning between grain collision forces and viscous forces.

Experimental data and field observations indicate that high nonequilibrium pore pressure p commonly exists in the bodies of debris flows and pyroclastic flows, but not at surge fronts or deposit margins (Wilson and Head, 1981; Iverson, 1997; Major and Iverson, 1999). Therefore, Coulomb friction generally dominates flow resistance at surge fronts, and viscous resistance and grain collisions gain significance in flow interiors. In all parts of such flows, however, contributions of grain interactions to flow resistance result from shear stresses that are proportional to the intergranular normal stress σ, which varies with p. Assessment of fluid pressure distributions therefore constitutes a central problem in predictive modeling (Iverson and Denlinger, 2001; Denlinger and Iverson, 2001).

Grain-size sorting almost invariably occurs in granular mass flows and enhances development of nonuniform fluid pressures and surge fronts. Commonly, flows selectively sort large grains upward and small grains downward. Size segregation is most efficient in flows with low but nonzero values of N_{Sav} (Vallance and Savage, 2000). Because size segregation produces concentrations of large grains at the flow surface, where downstream velocities are typically the greatest (Fig. 1), large grains migrate preferentially to surge fronts. Concentrations of coarse clasts at surge fronts can also develop through density-driven segregation of pumice (Vallance, 1994) and through preferential entrainment of coarse clasts by advancing surge fronts (Suwa, 1988; Vallance and Scott, 1997). Coarse-grained surge fronts in which flow resistance is focused can lead to lateral flow instability that causes flow fingering and results in digitate deposits (Fig. 2; Pouliquen et al., 1997).

CONCLUSIONS

Behavior of granular mass flows depends on evolution of granular temperature and nonequilibrium pore-fluid pressure, and this evolution precludes assessment of flow rheology from steady-state experiments or static deposits. However, experiments indicate that intergranular shear stresses in granular mass flows are generally proportional to intergranular normal stresses, regardless of the granular temperature, pore-fluid pressure, or shear rate. Therefore, as a good approximation, intergranular shear stresses obey the Coulomb equation and depend linearly on flow depth, grain concentration, and pore-fluid pressure. Factors that influence evolution of fluid pressure consequently deserve emphasis in efforts to predict and interpret granular mass flows. Initial and boundary conditions, grain-size segregation, and changes in flow volume may be at least as important as mixture composition in determining fluid pressures and flow dynamics. Field work can most profitably focus on these aspects of granular mass flows rather than on speculative interpretations of rheology.

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