APPENDIX

Because the worldwide probability of a large earthquake appears to be constant in time (Michael 2011), the probability of the occurrence of a catastrophic earthquake in any short interval dt is thought to be $\alpha N(t)dt$, where $N(t)$ is some measure of the population at risk at time t and α is a constant. The occurrence of catastrophic earthquakes is then a nonstationary Poisson process (see Ogata 1983 for a brief discussion of nonstationary Poisson events and the maximum likelihood method of analyzing them). Given a sequence of n catastrophic earthquakes at times $t_1,\ t_2,\ \ldots,\ t_n$ within the interval (S, T), the likelihood of that particular sequence is the product of the likelihoods $\alpha N(t_i)$ of an earthquake at each time t_i and the likelihoods $Exp[-\int_{t_i}^{t_{i+1}} \alpha N(t)dt]$ of no events in time intervals S to t_1, t_i to t_{i+1}, and t_n to T. Thus, the likelihood of the particular sequence is

$$L = \left(\prod_{i=1}^{n} \alpha N(t_i)\right) Exp\left(-\int_S^T \alpha N(t)dt\right) \tag{A1}$$

The natural logarithm of the likelihood is

$$Log_e(L) = nLog_e(\alpha) + \left(\sum_{i=1}^{n} Log_e(N(t_i))\right) - \alpha\int_S^T N(t)dt \tag{A2}$$

One can then can solve the equation $\partial(Log_e(L))/\partial\alpha = 0$ for the value

$$\alpha = \frac{n}{\int_S^T N(t)dt} \tag{A3}$$

that maximizes the likelihood of the particular sequence $t_1,\ t_2,\ \ldots,\ t_n$. The standard deviation σ_α of α is

$$\sigma_\alpha = \sqrt{\frac{\alpha}{\int_S^T N(t)dt}} \tag{A4}$$

One can compare as a function of time the observed cumulative number of catastrophic earthquakes in the time interval (S, t) with the expected number $\alpha\int_S^t N(t)dt$ predicted at any time t by the nonstationary Poisson process. However, one should realize that the observed accumulation curve may wander from predicted curve because the abscissa of the observed cumulative number of events is composed of the sum of independent random intervals. Because the time of an event then depends upon the time of the preceding event, the events are not independent. For example, if one of the early intervals between events is unusually long, the accumulation curve will fall below the predicted curve, and there is no reason for the subsequent events to compensate for that shortfall because the intervals are independent. The accumulation curve may then wander from the predicted curve in a random walk.

The Pearson's chi-square test (Evans 1955, p. 775) provides a quantitative measure of how well the data fit the proposed nonstationary Poisson distribution. Divide the time interval (S, T) into m subintervals such that a significant number (>5) of events is expected in each subinterval. Then the chi-square statistic (χ^2) is

$$\chi^2 = \sum_{i=1}^{m} \frac{(O_i - E_i)^2}{E_i} \tag{A5}$$

where O_i is the number of observed events lying in the ith subinterval, and $E_i = \alpha \int_{t_i}^{t_{i+1}} N(t)dt$ is the expected number of events in that subinterval. The probability P that a smaller value of χ^2 would be observed if the data were drawn from the proposed nonstationary Poisson process is given by the chi-square distribution with m-2 degrees of freedom and argument χ^2. A probability between 5% and 95% indicates that the data are consistent with proposed nonstationary Poisson distribution.

REFERENCES

Evans, R. D., 1955. The Atomic Nucleus, Tata McGraw-Hill Publishing Company, Bombay, India, 1002 pp., available at http://www.archive.org/details/atomicnucleus032805mbp.

Michael, A. J., 2011. Random variability explains apparent global clustering of large earthquakes, Geophysical Research Letters 38, L21301.

Ogata, Y., 1983. Estimation of the parameters in the modified Omori formula for the aftershock frequencies by the maximum likelihood procedure, *Journal of the Physics of the Earth* **31**, 115–124.