Map correlation method: Selection of a reference streamgage to estimate daily streamflow at ungaged catchments

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[1] Daily streamflow time series are critical to a very broad range of hydrologic problems. Whereas daily streamflow time series are readily obtained from gaged catchments, streamflow information is commonly needed at catchments for which no measured streamflow information exists. At ungaged catchments, methods to estimate daily streamflow time series typically require the use of a reference streamgage, which transfers properties of the streamflow time series at a reference streamgage to the ungaged catchment. Therefore, the selection of a reference streamgage is one of the central challenges associated with estimation of daily streamflow at ungaged basins. The reference streamgage is typically selected by choosing the nearest streamgage; however, this paper shows that selection of the nearest streamgage does not provide a consistent selection criterion. We introduce a new method, termed the map-correlation method, which selects the reference streamgage whose daily streamflows are most correlated with an ungaged catchment. When applied to the estimation of daily streamflow at 28 streamgages across southern New England, daily streamflows estimated by a reference streamgage selected using the map-correlation method generally provides improved estimates of daily streamflow time series over streamflows estimated by the selection and use of the nearest streamgage. The map correlation method could have potential for many other applications including identifying redundancy and uniqueness in a streamgage network, calibration of rainfall runoff models at ungaged sites, as well as for use in catchment classification.

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1. Introduction

[2] Daily streamflow time series are critical to a very broad range of hydrologic problems, including, but not limited to rainfall runoff model calibration and the determination of ecological needs for aquatic habitat. Daily streamflow time series are readily obtained from gaged catchments; however, streamflow information is commonly needed at catchments for which no measured streamflow information exists. Therefore, prediction of time series of daily streamflows at ungaged basins remains one of the central unsolved challenges in hydrology. This study focuses on improving the class of daily streamflow estimation methods that utilize a reference streamgage to estimate time series of streamflow at ungaged catchments.

[3] The most common and perhaps oldest method to estimate daily streamflow at an ungaged catchment by use of a reference streamgage is the drainage area ratio method, where daily streamflows on a given day t are estimated by

$$Q\mathrm{u}_t = \frac{A\mathrm{u}}{A\mathrm{g}} Q\mathrm{g}_t, \tag{1}$$

$Q\mathrm{u}_t$ is the streamflow on day t at the ungaged site, $Q\mathrm{g}_t$ is the streamflow on day t at the reference streamgage, $A\mathrm{u}$ is the drainage area of the ungaged catchment, and $A\mathrm{g}$ is the drainage area to the reference streamgage. As presented in this paper, equation (1) implies that the streamflow per unit area at the ungaged catchment and reference catchment are equal for any given time t.

[4] Another approach introduced by *Hirsch* [1979] is termed the maintenance of variance extension method. In this approach, standardized monthly streamflows at a reference streamgage are equated to the standardized monthly streamflows at the ungaged catchment. The monthly means and standard deviations used to standardize the monthly flows at the ungaged site were estimated by regional regression methods. This approach is readily extended to daily streamflow time series using the equation

$$Q\mathrm{u}_t = \hat{\mu}_\mathrm{u} + \hat{\sigma}_\mathrm{u}\left(\frac{Q\mathrm{g}_t - \hat{\mu}_\mathrm{g}}{\hat{\sigma}_\mathrm{g}}\right), \tag{2}$$

where $Q\mathrm{u}_t$ is the streamflow on day t at the ungaged site, $Q\mathrm{g}_t$ is the streamflow on day t at the reference streamgage, $\hat{\mu}_\mathrm{u}$ is the estimated daily mean streamflow at the ungaged catchment, $\hat{\mu}_\mathrm{g}$ is the daily mean streamflow at the reference streamgage, $\hat{\sigma}_\mathrm{u}$ is the estimated standard deviation of the daily streamflows at the ungaged catchment, and $\hat{\sigma}_\mathrm{g}$ is the standard deviation of the daily streamflows at the reference streamgage. Equation (2) implies that the standardized

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streamflow at the ungaged catchment and reference catchment for any given time t are equal.

[5] More recently, *Fennessey* [1994], *Smakhtin* [1999], *Smakhtin et al.* [1997], *Mohoamoud* [2008], and *Archfield et al.* [2010] applied a nonlinear spatial interpolation method between a reference streamgage and the ungaged catchment to estimate daily streamflow at an ungaged catchment. For this method, a daily flow-duration curve is first estimated for the ungaged catchment. The flow-duration curve is then assembled into a time series of daily streamflow by equating exceedence probabilities at the ungaged catchment to those at the reference streamgage. The dates associated with the corresponding exceedence probabilities at the reference streamgage are transferred to the ungaged catchment to obtain a time series of daily streamflow.

[6] In the methods presented above, the timing of the estimated streamflows at the ungaged catchment is entirely dependent upon the selection of the reference streamgage. These estimation methods assume that high-, low-, and mid-range flows at the ungaged catchment occur on the same day as they occur at the reference streamgage. If the daily streamflow time series at two catchments were perfectly correlated, all high-, low- and mid-range flow events would occur on the same days in each of the records, irrespective of the magnitude of the streamflows. In fact, correlation is the metric by which a reference streamgage is selected when these methods are used to extend and patch streamflow records (see *Smakhtin et al.* [1997] as an example).

[7] Because correlation between an ungaged catchment and a reference streamgage cannot be determined, the above cited methods recommend selecting the nearest streamgage [*Mohoamoud*, 2008] or several nearby streamgages [*Smakhtin*, 1999; *Smakhtin et al.*, 1997]. *Smakhtin* [1999] and *Smakhtin et al.* [1997] advocate using several nearby reference streamgages because one streamgage may not be adequate to represent the timing of the streamflows at the ungaged catchment. For the drainage area ratio method, *Hortness* [2006] recommends selecting a reference streamgage with a drainage area ratio between 0.5 and 1.5, again as an attempt to select a reference streamgage most representative of the timing of streamflows at the ungaged catchment.

[8] This paper first demonstrates that the selection of the nearest streamgage as a reference streamgage is not always the streamgage having the highest correlated daily streamflow values, even in a relatively homogeneous study region. Second, the paper shows that selection of a reference streamgage based on correlation between daily streamflow time series, rather than the distance between sites, substantially improves estimates of daily streamflows and that correlation between daily streamflow time series can be related to the goodness of fit of the estimated streamflows. Third, this paper introduces a new approach, which estimates the unbiased correlation between the time series of daily streamflows at an ungaged catchment and a potential reference streamgage. Last, this paper extends this new approach to the selection the reference streamgage having the highest correlation value with an ungaged catchment.

2. Database

[9] This paper utilizes daily streamflow values at 28 U.S. Geological Survey streamgages located in the New England region of the United States (Figure 1 and Table 1). The contributing drainage areas to these streamgages have been determined to have minimal human disturbances [*Armstrong et al.*, 2008]. Streamflow values at these streamgages have a common, 30 year period of record extending from 1 October 1967 through 30 September 1997. Streamflow values are all greater than zero, although the methods and results presented in this paper can be applied to basins with zero-value streamflows with a modification described in section 3.

[10] The rivers in this study region are of particular interest to various planners, managers, and stakeholder groups who are working to determine sustainable water withdrawal policies and instream-flow standards for this region. To support this process, *Archfield et al.* [2010] developed a decision support tool to evaluate water availability at ungaged sites by providing estimates of daily unregulated and regulated streamflow at a user-identified stream reach. This region was also the focus of recent studies to classify streamgages by streamflow statistics and climate and physical basin characteristics [*Armstrong et al.*, 2008].

3. Relation Between Distance and Correlation Among Streamflow Series

[11] This paper first examines how the distance between two streamgages is related to the correlation between the logarithms of the daily streamflow values at the streamgages. If there was a perfect relation between distance and correlation, then one could be reasonably assured that the choice of the nearest streamgage to an ungaged catchment would also be the streamgage having the highest correlated streamflow values.

[12] To explore the relation between distance and correlation, the Euclidian distance between each pair of streamgages was compared to r (Pearson's r correlation coefficient), computed from the logarithms of the daily streamflow values between the pairs of streamgages. Pearson's r correlation coefficient measures the linear correlation between two data series; therefore, the logarithms of the daily streamflow values were first taken to linearize the relation between the two streamflow time series. Although not used in this paper, one could use Kendall's τ to quantify the correlation between two streamflow time series. Kendall's τ is a nonparametric estimator of correlation that measures the monotonic (linear or nonlinear) relation between two data sets; because Kendall's τ is a rank-based metric, it can be computed for streamflow time series that contain zero-flow values [*Helsel and Hirsch*, 2002].

[13] The relations between distance and r for the study streamgages are shown in Figure 2. Approximately one third of the streamgages show a strong relation between distance and r; nonetheless, more than half of the streamgages show a weak or nearly no relation between distance and r (Figure 2). Interestingly, for more than two thirds of the streamgages, the nearest streamgage was not the streamgage having the highest r value (Table 2). For this region, distance cannot be considered a consistently good proxy for r even though the average distance between streamgages is only approximately 100 km.

[14] Although Table 2 shows that the nearest streamgage is not always the streamgage with the highest value of r, the streamgage with the highest r value typically was the next

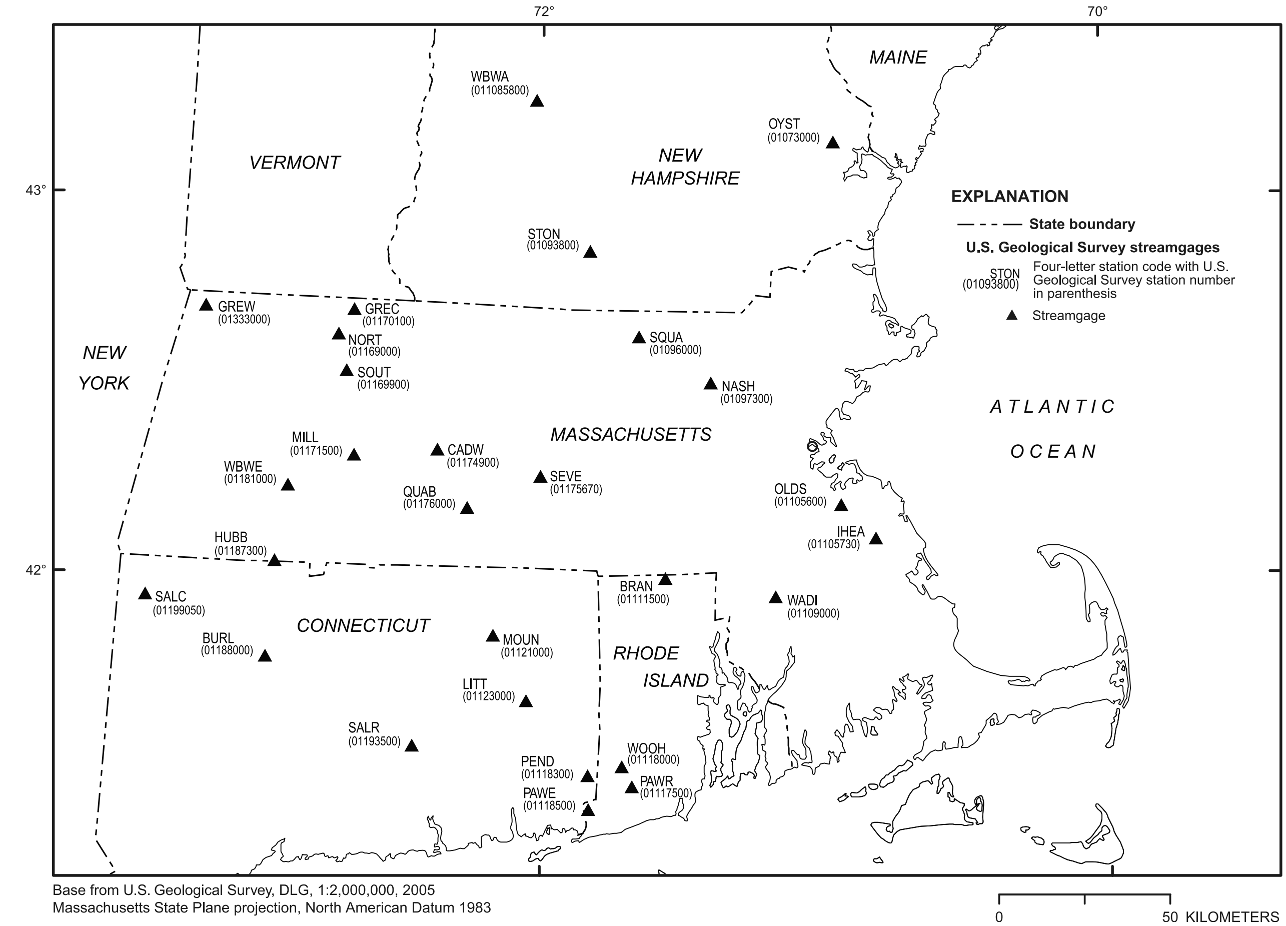

Figure 1. Locations of 28 U.S. Geological Survey least-altered streamgages in southern New England.

Table 1. Summary of 28 U.S. Geological Survey Least-Altered Streamgages in Southern New England

Streamgage Number	Four-Letter Abbreviation for Streamgage	Streamgage Name	Drainage Area (km^2)
01111500	BRAN	Branch River at Forestdale, RI	236.23
01188000	BURL	Burlington Brook near Burlington, CT	10.62
01174900	CADW	Cadwell Creek near Belchertown, MA	6.60
01170100	GREC	Green River near Colrain, MA	106.99
01333000	GREW	Green River at Williamstown, MA	109.92
01187300	HUBB	Hubbard River near West Hartland, CT	53.54
01105730	IHEA	Indian Head River at Hanover, MA	77.93
01123000	LITT	Little River near Hanover, CT	76.69
01171500	MILL	Mill River at Northampton, MA	139.81
01121000	MOUN	Mount Hope River near Warrenville, CT	70.24
01097300	NASH	Nashoba Brook near Acton, MA	33.02
01169000	NORT	North River at Shattuckville, MA	232.22
01105600	OLDS	Old Swamp River near South Weymouth, MA	11.34
01073000	OYST	Oyster River near Durham, NH	31.62
01118500	PAWE	Pawtucket River at Westerly, RI	761.22
01117500	PAWR	Pawcatuck River at Wood River Junction, RI	257.29
01118300	PEND	Pendleton Hill Brook near Clarks Falls, CT	10.39
01176000	QUAB	Quaboag River at West Brimfield, MA	387.13
01199050	SALC	Salmon Creek at Lime Rock, CT	76.59
01193500	SALR	Salmon River near East Hampton, CT	271.28
01175670	SEVE	Sevenmile River near Spencer, MA	22.79
01169900	SOUT	South River near Conway, MA	62.52
01096000	SQUA	Squannacook River near West Groton, MA	166.02
01093800	STON	Stony Brook tributary near Temple, NH	9.38
01109000	WADI	Wading River near Norton, MA	112.87
01085800	WBWA	West Branch Warner River near Bradford, NH	15.28
01181000	WBWE	West Branch Westfield at Huntington, MA	243.38
01118000	WOOH	Wood River Hope Valley, RI	192.10

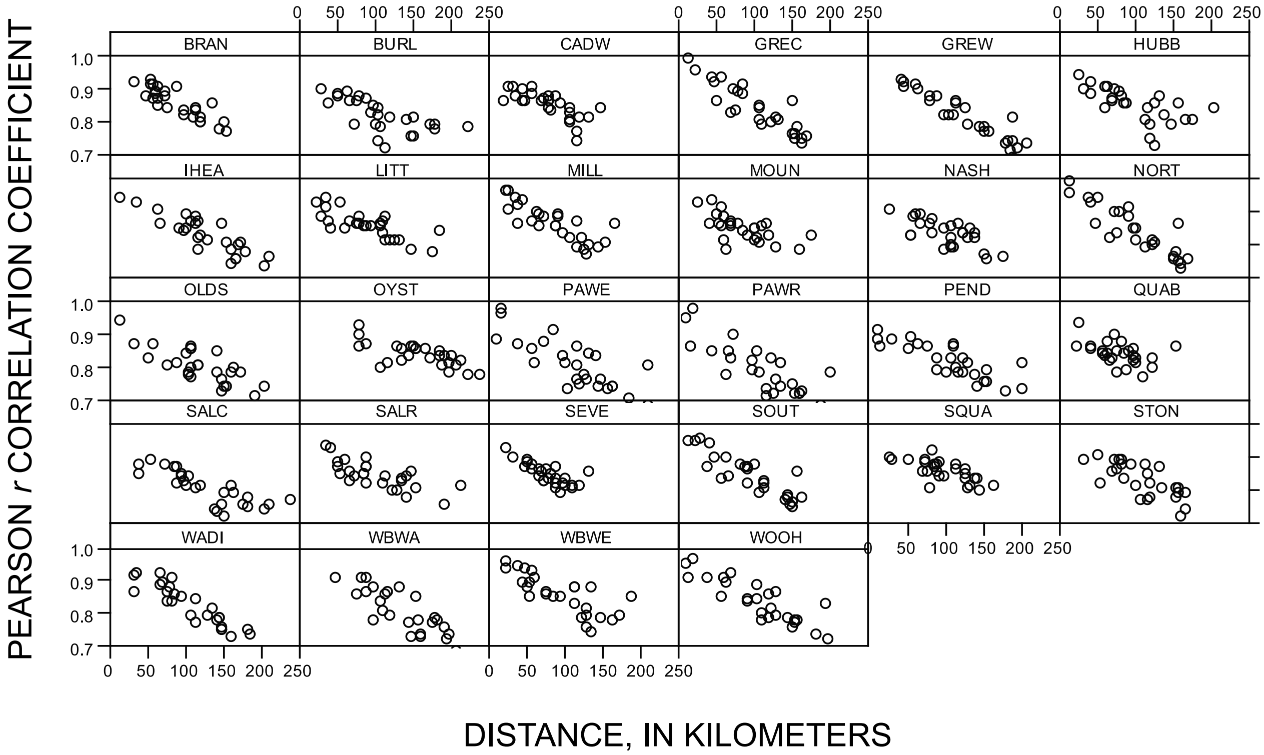

Figure 2. Relation between distance and Pearson's r correlation coefficient for 28 U.S. Geological Survey streamgages located in southern New England. Pearson's r correlation coefficient was computed from the logarithms of the daily streamflow values between each pair of streamgages using a 30 year period of record from 1 October 1967 through 30 September 1997.

nearest streamgage (Table 2 and Figure 1). Therefore, it was unclear what differences would result in the estimated daily streamflows if the streamgage with the highest r value was selected as the reference streamgage instead of the nearest streamgage. To compare the two selection criteria for reference streamgage (the nearest streamgage versus the most correlated streamgage), the drainage area ratio method (equation (1)) was applied to each of the study streamgages using both the nearest streamgage as the reference streamgage and the most correlated streamgage as the reference streamgage (the streamgage having the highest value of r). To evaluate goodness of fit, the Nash-Sutcliffe efficiency value E [*Nash and Sutcliffe*, 1970] was computed from both the observed and the estimated arithmetic and log-transformed streamflows (Table 3). Because streamflows at a streamgage may vary by orders of magnitude, the E value computed from the arithmetic streamflows may be not be representative of the goodness of fit across all streamflow values; hence, the E values computed from the log-transformed streamflow values provide a more representative measure of goodness of fit across the entire range of streamflow values with an emphasis on the fit for low streamflow values. *Oudin et al.* [2006] provide a discussion on the use of E in evaluating goodness of fit for high and low streamflow values.

[15] The median E value computed from both the estimated arithmetic and log-transformed streamflow values was higher when the streamgage with the highest r value was selected as the reference streamgage (Figures 3a and 3c). For cases where the nearest reference streamgage was not the reference streamgage with the highest r value, the selection of the reference streamgage with the highest r value always resulted in a higher E value than the selection of the nearest reference streamgage (Figure 3B), with the exception of two streamgages, and only for E values computed from the log-transformed streamflows (Figure 3d). For the PAWE streamgage, the improvement is quite dramatic (Figures 3b and 3d) even though the nearest streamgage (PEND) and highest correlated streamgage (PAWE) are separated by nearly the same distance (Figure 1 and Table 3); this same observation can be made of the QUAB streamgage (Figures 1, 3b, and 3d).

[16] It is not unexpected that the reference streamgage with the highest r value would outperform the use of the nearest reference streamgage. If the streamflows at the reference streamgage were perfectly correlated with the ungaged catchment, the only error in the streamflow estimates would arise from the manner in which the magnitudes of the streamflows were calculated. What was unexpected was the dramatic improvement that was made in the drainage area ratio method by selecting the reference streamgage with the highest r value. Because the other reference streamgage-based methods also transfer the timing of the streamflows at the reference streamgage to the ungaged catchment, we can infer that the same improvement would occur when the reference streamgage was selected on the basis of r rather than distance. Interestingly, only two streamgages having reference streamgages with the highest r values (STON and WADI) also had drainage area ratios that were within the 0.5−1.5 range suggested by *Hortness* [2006], although estimated streamflows agreed well with observed streamflows (Tables 2 and 3).

Table 2. Closest Reference Streamgage and Reference Streamgage Having the Streamflows Most Correlated to 28 Streamgages Located in Southern New England[a]

Study Streamgage	Nearest Streamgage	Distance Between the Study Streamgage and the Nearest Streamgage (km)	Most Correlated Streamgage	Pearson's r Correlation Coefficient Between Streamflows at the Most Correlated Reference Streamgage and the Study Streamgage
BRAN	WADI	156.15	LITT	0.93
BURL	***HUBB***	223.85	***HUBB***	0.89
CADW	QUAB	146.23	MILL	0.91
GREC	***NORT***	166.22	***NORT***	0.99
GREW	***NORT***	206.79	***NORT***	0.92
HUBB	***WBWE***	203.70	***WBWE***	0.94
IHEA	***OLDS***	213.43	***OLDS***	0.94
LITT	MOUN	186.88	SALR	0.94
MILL	WBWE	166.96	SOUT	0.96
MOUN	LITT	175.52	SALR	0.93
NASH	***SQUA***	175.81	***SQUA***	0.90
NORT	***GREC***	167.69	***GREC***	0.99
OLDS	***IHEA***	204.31	***IHEA***	0.94
OYST	STON	240.12	SQUA	0.93
PAWE	PEND	208.88	PAWR	0.98
PAWR	WOOH	203.51	PAWE	0.98
PEND	PAWE	199.26	WOOH	0.91
QUAB	CADW	151.19	SEVE	0.94
SALC	HUBB	240.12	WBWE	0.90
SALR	***LITT***	215.40	***LITT***	0.94
SEVE	***QUAB***	129.96	***QUAB***	0.94
SOUT	NORT	161.84	MILL	0.96
SQUA	NASH	162.28	OYST	0.93
STON	SQUA	163.92	WBWA	0.91
WADI	BRAN	186.74	WOOH	0.93
WBWA	STON	208.88	NORT	0.92
WBWE	***MILL***	187.82	***MILL***	0.96
WOOH	PAWR	197.28	PAWE	0.97

[a]Pearson's r correlation coefficient was computed from the natural logarithms of the daily streamflow values. Bolded, italicized reference streamgages show cases for which the nearest reference streamgage was also the most correlated reference streamgage.

[17] For the study streamgages that had lower E values, it was hypothesized that those E values resulted from the lack of a highly correlated streamgage present in the study region. If there was a relation between r and E, this relation could be used to infer information about the uncertainty in estimated streamflows at an ungaged catchment. To answer this question, E values were compared to the distance and r values corresponding to the respective reference streamgages selected in Table 2 (Tables 2 and 3 and Figure 4). Only E values computed from the log-transformed streamflows are shown. For this study area, E values greater than approximately 0.9 resulted when the reference streamgage had an r value greater than 0.95; however, no relation was observed between E and the distance between the study streamgage and the nearest reference streamgage (Figure 4). Therefore, unlike distance, r can also provide information about the certainty with which streamflow values may be estimated at an ungaged catchment using the drainage-area ratio method.

4. Map-Correlation Method

[18] On the basis of the comparisons in section 3, the certainty of estimates of daily streamflows at an ungaged site depends critically upon our ability to select a reference streamgage whose streamflows are most correlated with those at an ungaged catchment. The remainder of this paper introduces a new method, the map-correlation method, and evaluates its ability to estimate the correlation between a streamgage and an ungaged catchment. If the correlations between a set of streamgages and the ungaged catchment could be reliably estimated, one could then select the most correlated streamgage as the reference streamgage.

[19] The map correlation method is based on geostatistics. The use of geostatistics in hydrology is well established; however, applications to surface water hydrology are fewer than to groundwater problems. Surface water applications commonly involve the use of geostatistics to estimate a particular streamflow statistic or value at an ungaged location. For example, *Skøien and Blöschl* [2007] use geostatistical techniques to produce hourly streamflow estimates at an ungaged location by weighting observations at streamgages within the study area. Other works by *Villeneuve et al.* [1979], *Skøien and Blöschl* [2006], *Sauquet* [2006], *Chokmani and Ouarda* [2004], and *Sauquet et al.* [2008] use geostatistical techniques to obtain estimates of streamflow time series or particular streamflow statistics, such as mean annual and monthly streamflow or, in the case of *Horn* [1988], to estimate skew and autocorrelation of annual streamflows. In these cases, a geostatistical approach was used to directly weight observed streamflow values at streamgages to obtain estimates of these variables at an ungaged location.

[20] Similar to that of *Skøien and Blöschl* [2007], this paper follows the conceptual model described by *Woods and Sivapalan* [1999] that assumes that runoff exists at

Table 3. Nash-Sutcliffe Efficiency Values Obtained From Application of the Drainage-Area Ratio to 28 Streamgages in Southern New England by Selection of the Nearest Streamgage and the Streamgage Having the Most Correlated Streamflows to the Study Streamgage

Study Streamgage	Nash-Sutcliffe Efficiency Value Computed From the Arithmetic Observed and Estimated Streamflows		Nash-Sutcliffe Efficiency Value Computed From the Log-Transformed Observed and Estimated Streamflows	
	Choice of Nearest Reference Streamgage	Choice of Most Correlated Reference Streamgage	Choice of Nearest Reference Streamgage	Choice of Most Correlated Reference Streamgage
BRAN	0.76	0.73	0.64	0.85
BURL	0.68	0.68	0.39	0.39
CADW	0.39	0.71	0.72	0.81
GREC	0.92	0.92	0.98	0.98
GREW	0.39	0.39	0.84	0.84
HUBB	0.84	0.84	0.85	0.85
IHEA	0.44	0.44	0.85	0.85
LITT	0.80	0.82	0.74	0.82
MILL	0.62	0.77	0.88	0.92
MOUN	0.84	0.83	0.83	0.87
NASH	0.68	0.68	0.77	0.77
NORT	0.94	0.94	0.98	0.98
OLDS	0.73	0.73	0.88	0.88
OYST	0.30	0.78	0.77	0.77
PAWE	−0.34	0.96	0.06	0.96
PAWR	0.71	0.96	0.89	0.95
PEND	0.55	0.69	0.66	0.67
QUAB	−1.20	0.52	0.49	0.68
SALC	−0.50	−0.33	0.34	0.66
SALR	0.82	0.82	0.87	0.87
SEVE	0.73	0.73	0.82	0.82
SOUT	0.79	0.85	0.91	0.92
SQUA	0.72	0.74	0.62	0.62
STON	0.56	0.62	0.79	0.81
WADI	0.61	0.79	0.77	0.65
WBWA	0.69	0.74	0.83	0.77
WBWE	0.80	0.80	0.91	0.91
WOOH	0.81	0.87	0.91	0.93

any location. Therefore, we assume that it is possible to estimate the cross-correlation between the daily streamflows at a streamgage and an ungaged site for any location in the study area. The map-correlation method does not assume that the study area is limited by regions of homogeneity or other hydrologic boundaries. The implementation of the map-correlation method requires some minimum number of streamgages in the network for the geostatistical methods to be reliable; however, this study does not address that issue. The records at each of the streamgages must be coincident with the other streamgages in the study area for a long enough period such that estimates of correlation are representative of the full range of daily streamflow values for the streamgages in the study region. We further assume that the r values are isotropic across the study region.

[21] Figure 2 summarizes estimates of the correlation between each study streamgage and each of the other streamgages for our study region. These values are illustrated geographically in Figure 5 for one particular gage (OLDS, see Table 1 for site information), with an r value associated with each of the streamgages. The r values shown in Figure 5 are Pearson's r correlation coefficient values estimated from the logarithms of the daily streamflow values at the OLDS streamgage and at each of the other respective streamgages in the study region. For example, the r value estimated between the streamflows at the OLDS streamgage and the IHEA streamgage is 0.94. One can determine a relation between the semivariance, the squared differences between r for each pair of streamgages in the study region, and the corresponding separation distance (the Euclidian distance between each pair the streamgages) between each pair of streamgages. When plotted, this relation results in a variogram cloud [*Isaaks and Srivastava*, 1989]. The term cloud is used because the relation between semivariance and separation distance usually has no discernable pattern containing $\binom{n}{2}$ points, where n equals 27 for this study. Following standard geostatistical methods presented by *Isaaks and Srivastava* [1989], points in the variogram cloud can be binned within specified separation distances of one another to obtain a sample variogram, the relation between semivariance and separation distance. A continuous function relating semivariance and separation distance, termed the variogram model, is then fit to the sample variogram.

[22] For this study, the semivariance value for each resulting bin is computed as the average of the semivariance values that fall within each bin. This binning process results in a sample variogram that is used to fit the variogram model. The length and number of bins were determined by trial and error through visual plotting of the sample variograms and fitted models. After the trial and error period, the same bin length was used to fit the variogram model for each of the 28 streamgages. The R statistical program [*R Development Core Team*, 2005] and the related geoR software package [*Ribeiro and Diggle*, 2001] were used to bin the variogram clouds, generate the sample variograms, and fit the variogram model. The variogram model was fit using weighted least squares, with the weights determined by the number of points used to estimate each position on the sample variogram [*Ribeiro and Diggle*, 2001]. A spherical variogram model was fit for each study streamgage because of its relatively simple formulation and its visual

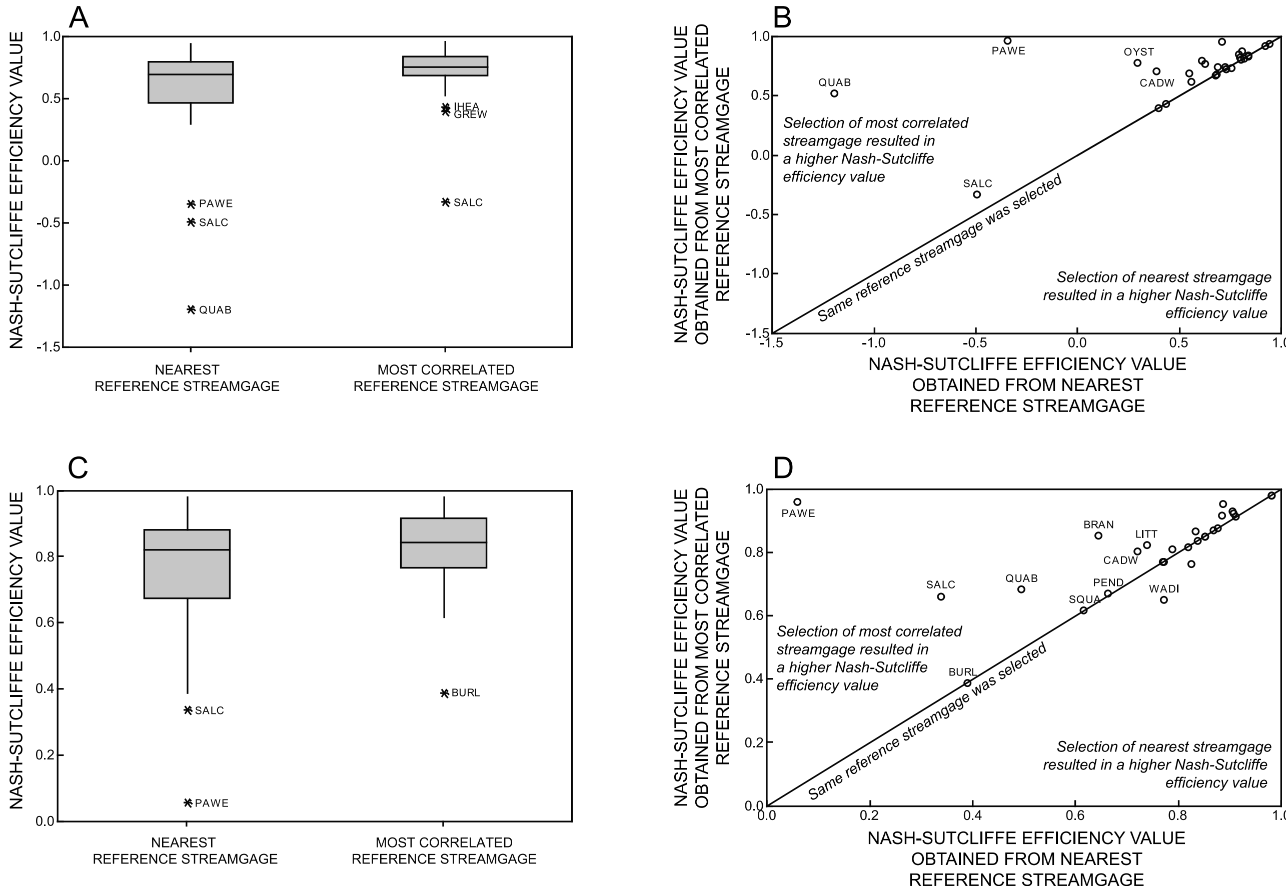

Figure 3. (a and c) Range of Nash-Sutcliffe efficiency values for daily streamflow values estimated at 28 U.S. Geological Survey streamgages using the drainage-area ratio method. (b and d) Comparison of Nash-Sutcliffe efficiency values based on the criteria used to select the reference streamgage. (a and b) Nash-Sutcliffe efficiency values estimated from the arithmetic values of observed and estimated streamflows. (c and d) Nash-Sutcliffe efficiency values estimated from log-transformed values of observed and estimated streamflows.

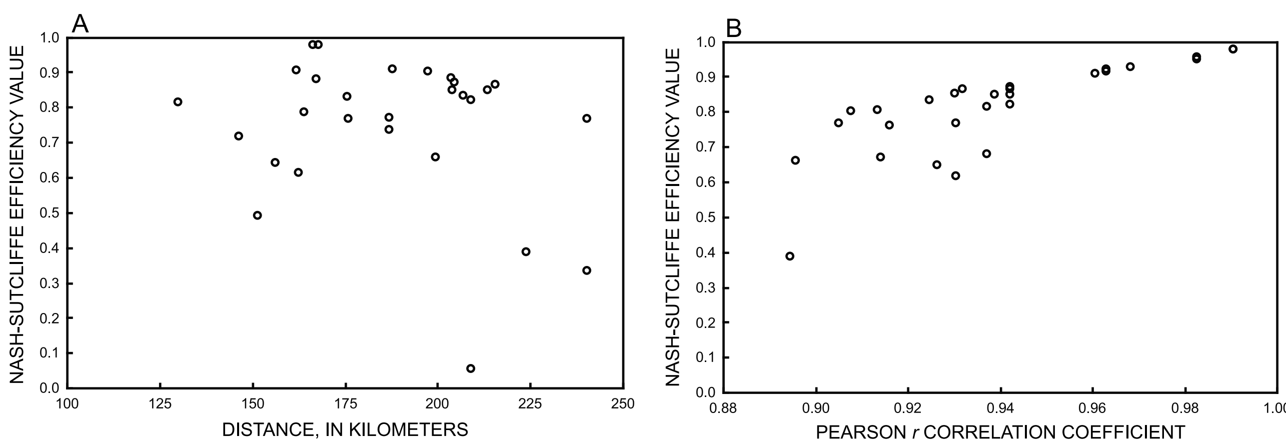

Figure 4. (a) Relation between Nash-Sutcliffe efficiency values obtained from application of the drainage-area ratio to 28 study streamgages in southern New England to distance between the study streamgage and the nearest streamgage and (b) relation between Nash-Sutcliffe efficiency values obtained from application of the drainage-area ratio to 28 study streamgages in southern New England to distance between the study streamgage and the nearest streamgage. Nash-Sutcliffe efficiency values are computed from log-transformed observed and estimated daily streamflow values.

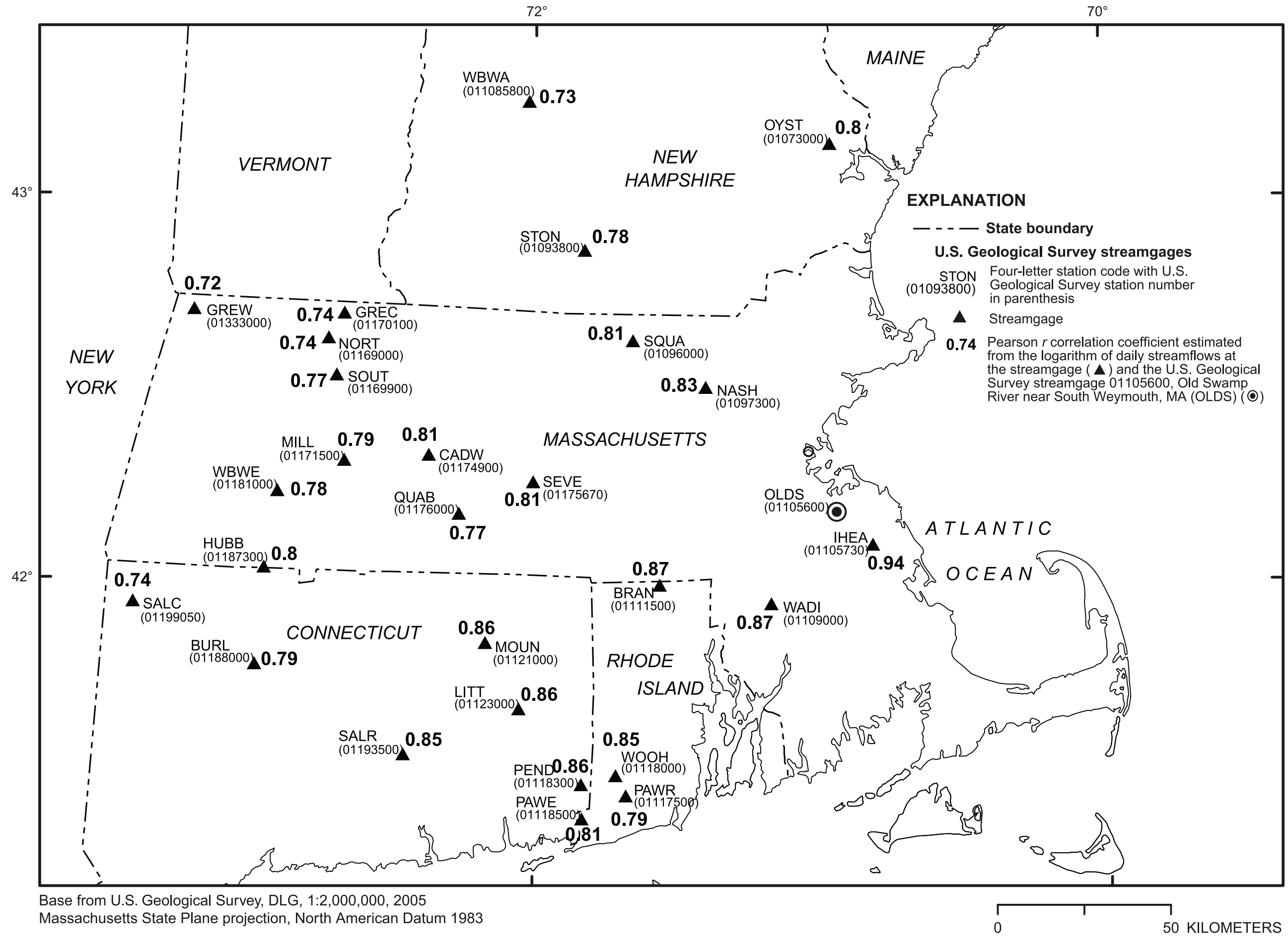

Figure 5. Pearson's r correlation coefficient values computed between the logarithms of the daily streamflow flow time series at the OLDS streamgage and 27 other streamgages in southern New England.

agreement with the majority of the sample variograms. The spherical variogram, as presented in the study by *Ribeiro and Diggle* [2001], has the form

$$\gamma(h) = \begin{cases} 1 - 1.5\dfrac{h}{a} + 0.5\left(\dfrac{h}{a}\right)^3 & h < 0 \\ 0 & \text{otherwise} \end{cases}, \quad (3)$$

where $\gamma(h)$ is the variogram model (also referred to as the correlation function), h is the separation distance, and a is the range parameter. Following from traditional geostatistics techniques for ordinary kriging as presented by *Isaaks and Srivastava* [1989], when $\gamma(h)$ (equation (3)) is multiplied by the partial sill, σ^2, the covariance function, $C(h)$, is obtained by [*Ribeiro and Diggle*, 2001]

$$C(h) = \sigma^2\gamma(h). \quad (4)$$

[23] Parameters of the spherical variogram models were estimated for each of the 28 study streamgages. Estimated variogram model parameters are shown in Table 4. The parameters at each study streamgage, when used in conjunction with equations (3) and (4) estimate the covariance between two r values at any distance apart from one another [*Ribeiro and Diggle*, 2001].

[24] To determine if a spatial covariance structure in the r values exists, a leave-one-out cross-validation experiment using the geoR package [*Ribeiro and Diggle*, 2001] was conducted for each of the 28 variogram models. Recall that for each of the 28 study streamgages, there are 27 r values with a corresponding variogram model fitted from the relation between the semivariance and separation distances determined from the $\binom{27}{2}$ pairs of streamgages. For each of the 28 variogram models (Table 4), each of the 27 r values were systematically removed from the sample variogram and the variogram model parameters were reestimated. For each removed streamgage, the r value for the removed streamgage was estimated from the variogram model and compared to the actual value. The E value was computed from the 27 estimated and observed values of r to provide a measure of goodness of fit for each of the 28 variogram models. The range of E values obtained from the cross-validation experiment is shown in Figure 6. The median E value is 0.76, and the average E value is 0.70 with a minimum and maximum value of 0.18 and 0.94, respectively. The sample variogram, fitted variogram models, and the observed and estimated r values from the cross-validation experiment are shown in Figure 7 for the highest, average, and lowest E values.

[25] The variogram models and cross-validation results indicate strong spatial structure between the

Table 4. Spherical Variogram Model Parameters Estimated From Pearson's r Correlation Coefficient Computed Between Streamflows at Each of 28 Study Streamgages in Southern New England and the Other 27 Remaining Streamgages

Study Streamgage	Variance Parameter, σ^2	Range Parameter, a
BRAN	0.00295	152,727.3
BURL	0.00323	152,727.3
CADW	0.00237	152,727.3
GREC	0.00841	191,131.6
GREW	0.00850	205,266.6
HUBB	0.00366	152,727.3
IHEA	0.00472	152,727.3
LITT	0.00227	152,727.3
MILL	0.00434	152,727.3
MOUN	0.00170	61,092.9
NASH	0.00218	152,727.3
NORT	0.00856	175,372.8
OLDS	0.00370	152,727.3
OYST	0.00169	152,727.3
PAWE	0.00948	196,811.6
PAWR	0.00949	189,778.8
PEND	0.00385	160,808.1
QUAB	0.00084	91,636.4
SALC	0.00379	152,727.3
SALR	0.00232	152,727.3
SEVE	0.00128	61,090.9
SOUT	0.00559	152,727.3
SQUA	0.00134	152,727.3
STON	0.00383	152,727.3
WADI	0.00575	172,058.2
WBWA	0.00751	156,072.1
WBWE	0.00505	152,727.3
WOOH	0.00599	152,727.3

r values for a given study streamgage. For most streamgages, it is also possible to reliability estimate r using geostatistics; however, Figure 7 indicates some streamgages have a stronger spatial structure than others. It is interesting to observe that the locations of the streamgages do not change for each of the variogram models; the only difference in the models is due to differences in the semivariance. Potential reasons for the differences in variogram model fits and the spatial structure of r across the study streamgages are left for future study; however, one could speculate from Figure 7 that the magnitudes of the semivariance values play some role in the utility of the variogram model to estimate r.

[26] The variogram model parameters and covariance function together describe the continuous spatial relation between the separation distance and the r values for each study streamgage. From this information, ordinary kriging can be used to estimate the unbiased value of r between any ungaged location and each of the streamgages in the study region. For a given study streamgage, the covariance function and estimated parameters are applied to each entry in a 28 × 28 matrix of separation distances, with separation distances computed between each pair of the 28 streamgages [*Isaaks and Srivastava*, 1989]. This matrix is termed the covariance matrix. Separation distances are also computed between the outlet to the ungaged catchment and each of the 28 streamgages. The covariance function is also applied to these separation distances, and the values are placed in a 28 × 1 matrix [*Isaaks and Srivastava*, 1989]. Ordinary kriging multiplies the transposed covariance matrix by the 28 × 1 matrix of covariances between the ungaged catchment and the 28 streamgages to obtain a set of 28 weights [*Isaaks and Srivastava*, 1989]. These weights are multiplied by each of the 28 respective r values to obtain the unbiased, minimum variance estimate of the correlation between the given study streamgage and the ungaged catchment [*Isaaks and Srivastava*, 1989]. A full description of the method to obtain an estimate of a variable of interest using kriging is given by *Isaaks and Srivastava* [1989].

[27] By applying the ordinary kriging procedure described above, it is possible to obtain a continuous map of correlation between a given streamgage and the entire study region. These correlation maps show the spatial distribution of the correlation between a given streamgage and any other location in the study region. Two examples of such maps are shown in Figures 8 and 9 for the OLDS and BRAN streamgages, respectively (see Table 1 for information on sites). These maps show the remarkably complex spatial structure associated with the r values. Correlation with the OLDS streamgage appears to decrease radially from the IHEA streamgage (Figure 8); this is an expected result because, for the OLDS streamgage, the nearest streamgage (IHEA) is also the most correlated streamgage (Table 2). In contrast, the spatial distribution of correlation with the BRAN streamgage is not as straightforward. There are four streamgages (WADI, SEVE, MOUN, and LITT) in close proximity to the BRAN streamgage (Figure 9); however, the map shows correlation decreases in an elliptical rather than radial pattern from the BRAN streamgage. It is also clear from the map that the nearest streamgage (WADI) is not the most correlated streamgage (LITT) (Table 2). The correlation maps show that correlation between two streamgages can be an extremely complex process that is not well represented by the proximity between two streamgages alone.

5. Application to Ungaged Catchments

[28] Previous sections show that a strong spatial relation between r values was observed for the study region and that

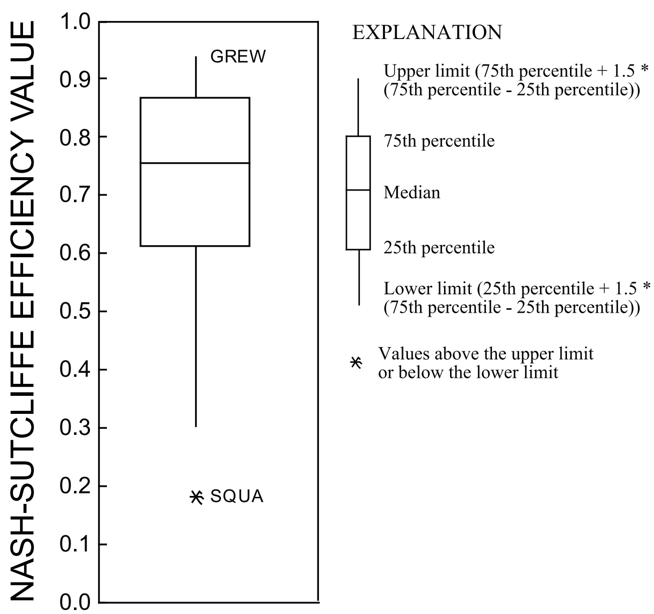

Figure 6. Range of Nash-Sutcliffe efficiency values resulting from the leave-one-out cross-validation of 28 spherical variogram models.

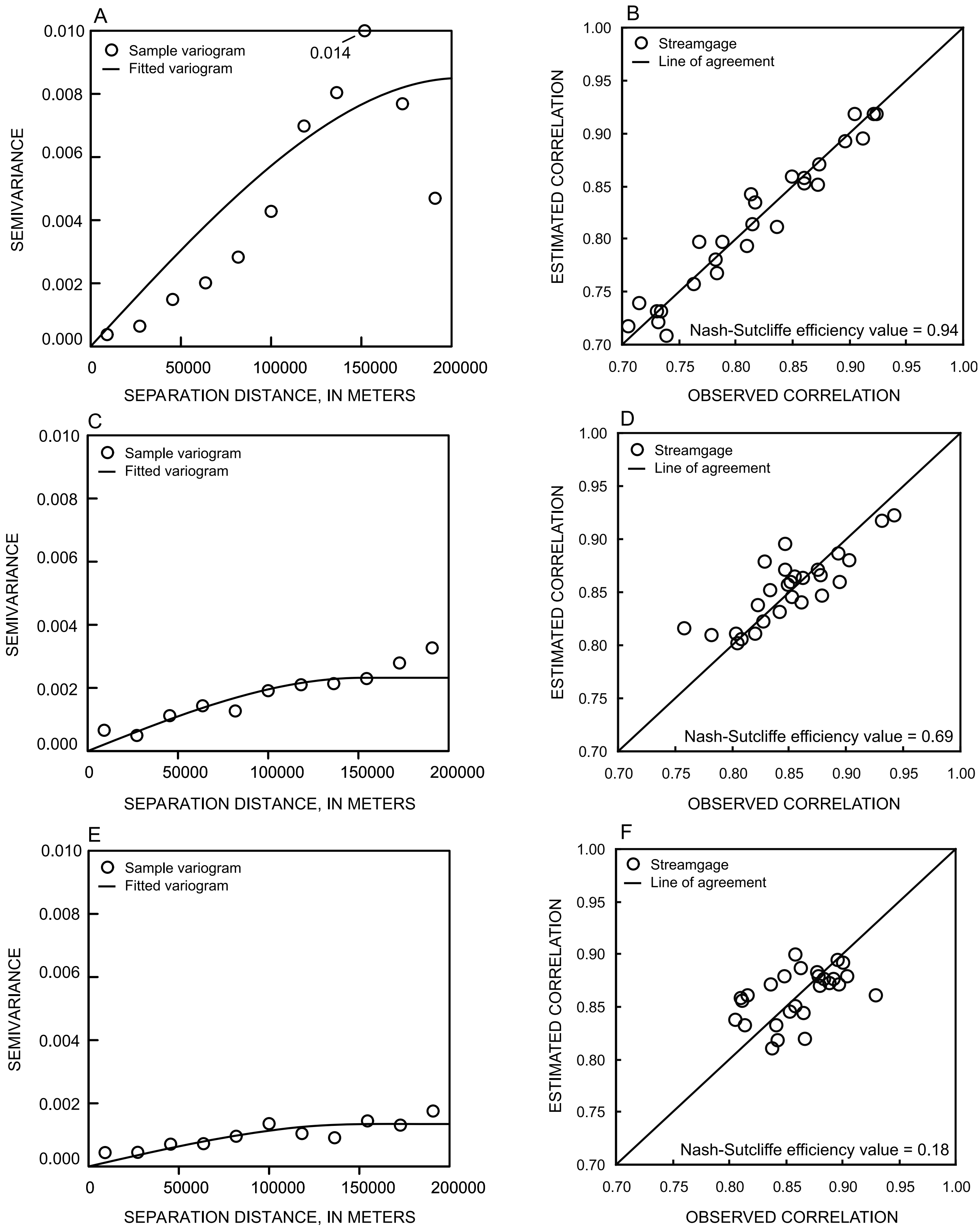

Figure 7. Sample and model variograms used to model the spatial structure of Pearson's *r* correlation coefficient with the (a) GREW, (c) SALR, and (e) SQUA streamgages. (b, d, and f) Observed and estimated correlations resulting from a leave-one-out cross-validation of the respective variograms.

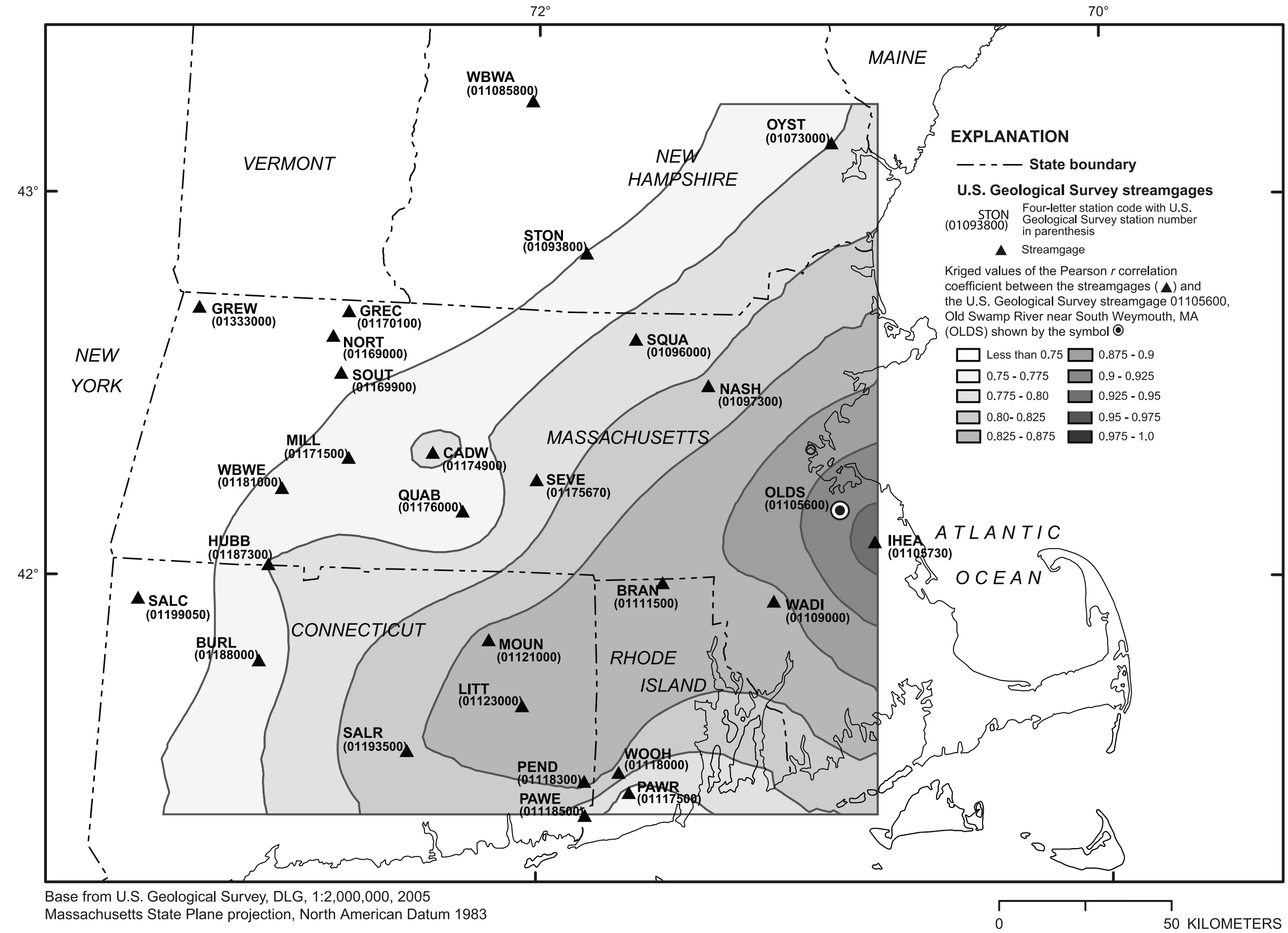

Figure 8. Map showing the spatial distribution of Pearson's r correlation coefficient values estimated between the daily streamflow time series at the OLDS streamgage and all other locations in southern New England.

this spatial relation could be reliably modeled for most of the study streamgages using ordinary kriging. This section evaluates the utility of the map-correlation method to select the most correlated reference catchment. In section 3, the correlations between the study streamgage and potential reference streamgages were estimated directly from the streamflow data; here we are interested in whether the map-correlation method can outperform the selection of the nearest reference streamgage when correlation cannot be directly estimated from available streamflow data.

[29] To address this hypothesis, each of the 28 study streamgages was assumed to be an "ungaged" catchment. The variogram models developed at each of the other 27 streamgages were used to estimate correlation between the "ungaged" catchment and the 27 potential reference streamgages. The "ungaged" catchment was removed from the sample variogram, and the variogram model parameters were re-fit to ensure that the "ungaged" catchment did not influence the selection of the reference streamgage. The estimated correlations between the each of the 27 potential reference streamgages were computed and the streamgage having the highest estimated r value was selected as the reference streamgage. The drainage area ratio method was then applied to the "ungaged" catchment. Just as in section 2, E values were computed from the arithmetic and log-transformed observed and estimated daily streamflow values at each of the "ungaged" catchments and compared to the E values obtained when the nearest streamgage is selected as the reference streamgage. The results of these comparisons are shown in Figure 10.

[30] For approximately one third of the study streamgages, the "ungaged" catchments, the map-correlation method correctly selected the most correlated streamgage as the reference streamgage (Table 5). For the streamgages where map-correlation did not select the most correlated reference streamgage, the difference in r values between the most correlated streamgage and the map-correlation-selected streamgage was, on average, 0.025 (Table 5).

[31] The map-correlation method yields similar results as in the earlier experiment, which used the observed correlations to select the reference catchment. In general, the E values resulting from the map-correlation-selected reference streamgage led to higher E values (Figures 10a and 10c) than when the closest streamgage was chosen. In fact, the majority of the E values are larger, in some cases substantially larger, than the E values resulting from the selection of the nearest reference streamgage (Figures 10b and 10d). For the streamgages that resulted in higher E values when the nearest reference streamgage was selected, the improvement in E values is not large (Figures 10b and 10d). It is important to note that, even for cases where the nearest reference streamgage outperformed the map correlation method, dis-

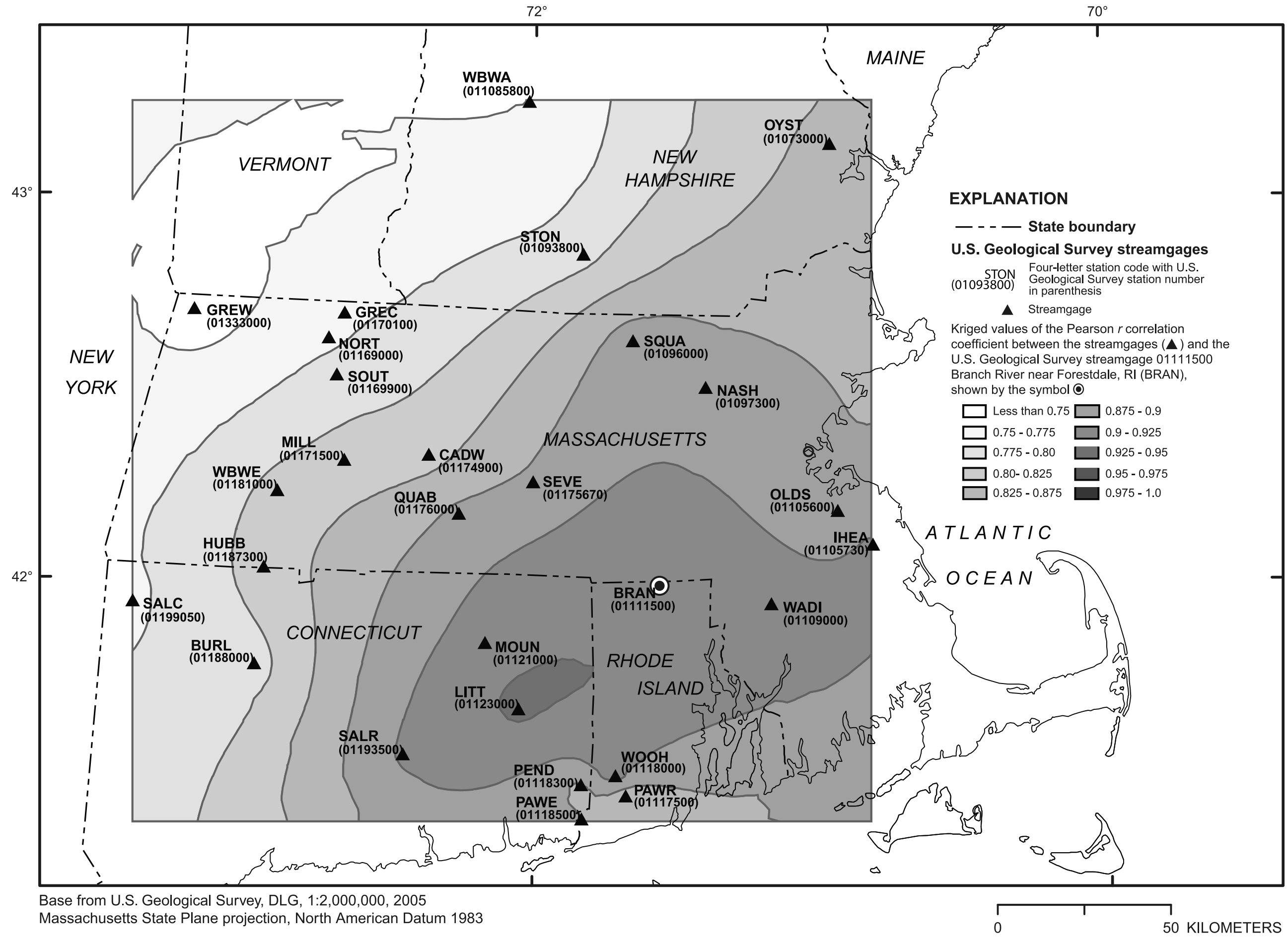

Figure 9. Map showing the spatial distribution of Pearson's r correlation coefficient values estimated between the daily streamflow time series at the BRAN streamgage and all other locations in southern New England.

tance does not provide a reliable framework to evaluate the uncertainty of the estimated streamflow values (Figure 4).

6. Discussion and Limitations

[32] This paper introduces the map correlation method, a method to estimate the correlation in daily streamflow between a streamgage and an ungaged location. Although used in this paper to estimate daily streamflow time series at ungaged catchments, the map-correlation method has potential application to many hydrologic problems, including calibration of hydrologic models, evaluation of streamgage networks and catchment classification. The map-correlation method illustrates the spatial distribution of correlation across a study region; thus, it may be used to identify redundancies and uniqueness in streamgage networks. *Wagener et al.* [2007] indicate a need for a common framework for hydrologic classification of rivers and streams that incorporates the complex interaction between climate and physical properties of basins. We conjecture that the correlation between time series of daily streamflow integrates the complex physical processes that govern streamflow. For example, if daily streamflow time series at two catchments are perfectly correlated, it is likely that the catchments are integrating the responses of their respective physical and climate characteristics in the same way, and thus the hydrologic responses of the catchments would be identical. Although map-correlation is used to select one reference streamgage, the method does not preclude the selection of more than one reference streamgage to be used in its application. For example, daily streamflows at the ungaged catchment could be weighted by the estimated correlation at each of the reference streamgages.

[33] There are limitations to the map-correlation method as implemented in this paper; however, future research may be able to overcome many of these limitations. To address situations with zero streamflow, the Kendall's τ measure of correlation could be used to estimate correlation in place of the Pearson's r correlation coefficient. Because it is a rank-based metric, Kendall's τ is insensitive to monotonic transformations of the data and can be used on zero-value streamflows [*Helsel and Hirsch*, 2002]. If the Pearson's r correlation coefficient is to be used, other transformations may avoid the problems encountered when streamflows with zero values are present in the record, such as a power transformation or the transformation shown in equation (9) of *Koutsoyiannis et al.* [2008].

[34] The density of the streamgage network and period of record used to estimate r values were not evaluated, although in the limit (few streamgages with short non-coincident periods of record), this would surely pose a severe restriction on the applicability of the method.

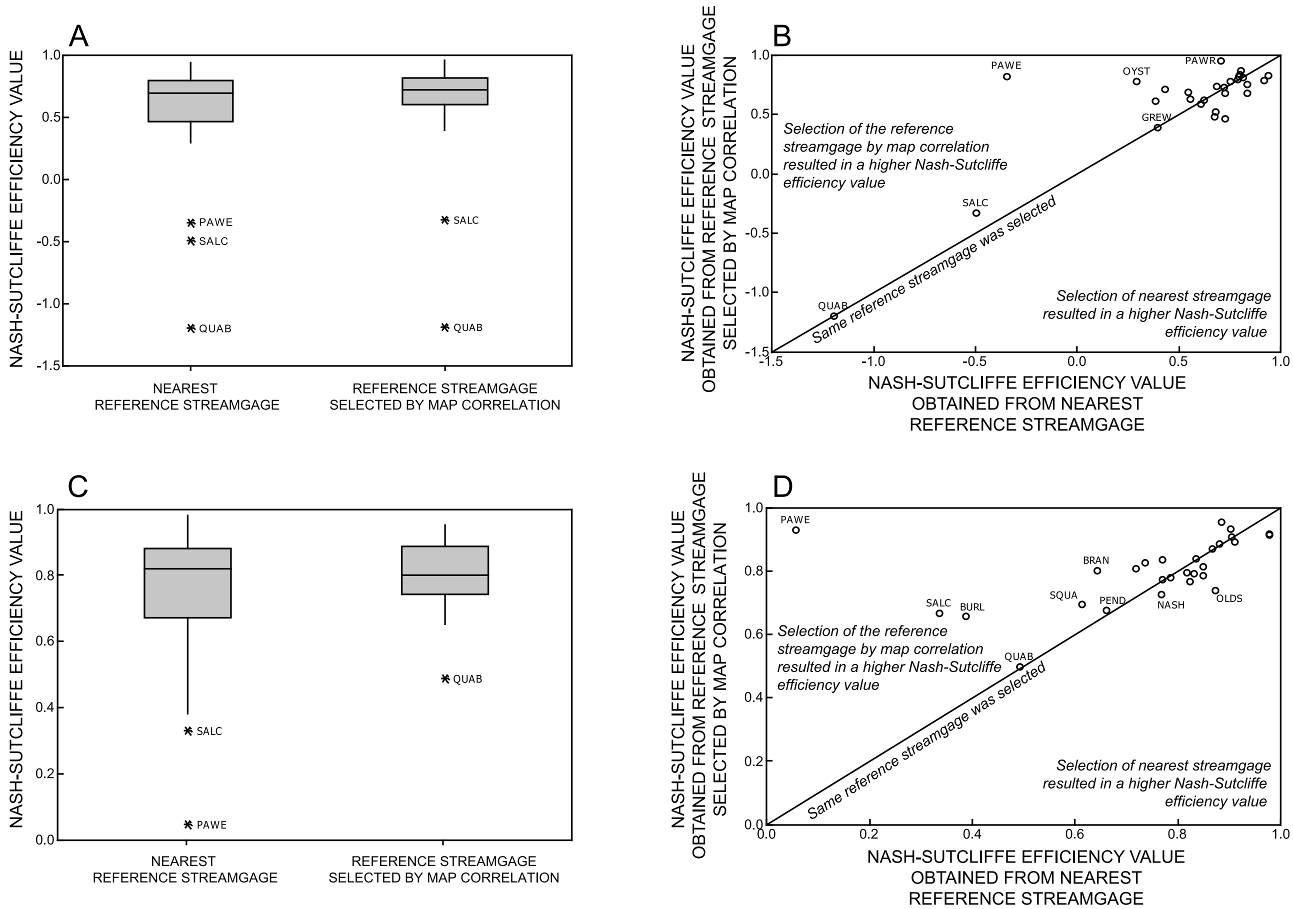

Figure 10. (a and c) Range of Nash-Sutcliffe efficiency values for daily streamflow values estimated at 28 U.S. Geological Survey streamgages using the drainage area ratio and map-correlation methods. (b and d) Comparison of Nash-Sutcliffe efficiency values based on the criteria used to select the reference streamgage. (a and b) Nash-Sutcliffe efficiency values estimated from the arithmetic values of observed and estimated streamflows. (c and d) Nash-Sutcliffe efficiency values estimated from log-transformed values of observed and estimated streamflows.

Although not tested for other geographic regions of larger and smaller sizes, methods to extend and patch records, which also use correlation between streamflows to identify a reference streamgage, have been in widespread use for over three decades; hence, once could reasonably assume that this method would be applicable to other study regions.

[35] Streamflow records were not tested for nonstationarity because the study streamgages were located in relatively undeveloped basins with minimal land cover or water use change over their respective periods of record. It is unclear what effect nonstationarity in the underlying daily streamflow records would have on the estimated correlations. If the relation between two streamflow time series is not constant or if nonlinear relations were present, the use of the Kendall's τ metric to quantify correlation may overcome these issues. The assumption of nonstationarity could be assessed in future applications of the map-correlation method. It is also unclear if the map-correlation method can be used at other time resolutions, such as subdaily, monthly, seasonal, or annual time scales.

[36] It may also be of interest to test other variogram models, including nonparametric models that do not require the assumption of arbitrary binning [*Gorsich and Genton*, 2000]. Other forms of kriging may also prove useful, such as universal kriging or cokriging, which could potentially blend similarities of basin characteristics into the kriging procedure.

7. Summary and Conclusions

[37] Methods to estimate daily streamflow time series at ungaged catchments typically require the use of a reference streamgage to transfer of the timing of the daily streamflows at the reference streamgage to the ungaged catchment. For this reason, the identification and selection of a reference catchment is one of the central challenges associated with estimation of daily streamflow time series at ungaged catchments. Our findings indicate that distance between a reference streamgage and ungaged catchment is not always a consistent and sensible selection criterion. The drainage area ratio method was applied to 28 streamgages in southern New England using both the nearest reference streamgage and the most correlated reference streamgage to estimate daily streamflows at these streamgage. Selection of the most correlated reference streamgage can, in some cases, dramatically improve the estimates of daily streamflow when compared to selection of the nearest reference streamgage.

Table 5. Reference Streamgage Having the Most Correlated Streamflows as Determined by the Map-Correlation Method for 28 Streamgages in Southern New England[a]

Study Streamgage	Most Correlated Streamgage, as Determined by the Map-Correlation Method	Difference in Pearson's *r* Correlation Coefficient Between the Most Correlated Streamgage and the Streamgage Selected by Map Correlation
BRAN	WOOH	0.016
BURL	WBWE	0.011
CADW	SEVE	0.004
GREC	SOUT	0.035
GREW	***NORT***	***0***
HUBB	MILL	0.016
IHEA	BRAN	0.036
LITT	***SALR***	***0***
MILL	WBWE	0.002
MOUN	BRAN	0.022
NASH	OYST	0.041
NORT	SOUT	0.033
OLDS	WADI	0.073
OYST	***SQUA***	***0***
PAWE	WOOH	0.014
PAWR	***PAWE***	***0***
PEND	***WOOH***	***0***
QUAB	CADW	0.075
SALC	***WBWE***	***0***
SALR	***LITT***	***0***
SEVE	SQUA	0.04
SOUT	GREC	0.006
SQUA	SEVE	0.034
STON	GREC	0.014
WADI	IHEA	0.001
WBWA	***NORT***	***0***
WBWE	SOUT	0.011
WOOH	***PAWE***	***0***

[a]Streamgages correctly identified as the most correlated streamgage are shown in bold, italicized font.

Furthermore, the use of the nearest reference streamgage had no relation to the goodness of fit of the estimated streamflows.

[38] The map correlation method was developed to provide an alternative selection criterion when choosing a reference catchment. The map-correlation method estimates the correlation between daily streamflow at an ungaged catchment and a potential reference streamgage by kriging the observed correlations between daily streamflow time series at a set of streamgages. Strong spatial relations between correlations across the study region were observed, and these relations were able to be successfully modeled through traditional geostatistical methods. Spatial models were used to estimate the correlation between an ungaged location and a set of streamgages and select the reference streamgage that resulted in the highest correlation value. The drainage area ratio was again applied to each of the 28 study streamgages using the map-correlation-selected reference streamgage. The map-correlation method generally provided improved estimates of daily streamflow time series over streamflows estimated from the selection and use of the nearest reference streamgage. The map-correlation method also has many other applications, including identifying redundancy and uniqueness in a streamgage network, calibrating hydrologic models at ungaged sites, and classifying catchments.

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