

## Saugus-Palmdale, California, Field Test for Refraction Error in Historical Leveling Surveys

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A field test designed to measure atmospheric refraction error in historical and modern leveling was conducted in May–June 1981 on a 50-km-long grade from Saugus to Palmdale, California. During 1955–1971, the length of sights made between the level instrument and rods systematically decreased from 60 m to 26 m. The difference in height near Palmdale measured by single-run long-sight (42-m) and short-sight (22-m) leveling during the test was 6 times larger than expected random error. Correction for refraction by using either the observed or modeled vertical temperature gradient in Kükkamaki's balanced sight equation reduced the height difference to the level of random error uncertainty. The observed temperature gradient obeyed a power law relation,  $T = a + bz^c$ , where  $z$  is the height and  $a$ ,  $b$ , and  $c$  are constants that depended on atmospheric conditions and the ground surface beneath the line of sight. The refraction-corrected leveling satisfies the specifications and meets all standards of first-order control surveys. The six historical surveys of the Saugus–Palmdale grade were corrected for refraction error using the results of the experiment and for rod scale errors and nontectonic subsidence considered in previous investigations. The corrected uplift near Palmdale reached  $56 \pm 16$  mm with respect to Saugus during the period 1955–1965. This amount of uplift is about one third that obtained before removal of refraction error. The corrected displacement profiles also reveal previously unrecognized deformation in the epicentral region of the 1971 San Fernando  $M_L = 6.4$  earthquake during the decade before the main shock.

### INTRODUCTION

We report on a field test of atmospheric refraction error designed to compare historical leveling from 1955–1964 with its modern successor. The results of this experiment, conducted jointly by the U.S. Geological Survey and the National Geodetic Survey (NGS) in 1981, are used to test the effectiveness of refraction error models on precise geodetic leveling. Holdahl [1982] reported briefly on preliminary findings of the experiment, Whalen and Strange [1983] presented a more detailed analysis of the results, Shaw and Smietana [1983] used the data to test a model for refraction error, and Castle *et al.* [1983b] and Craymer and Vaniček [this issue] examined the experiment for errors. In this report, we focus on the vertical temperature gradient that causes refraction, and on the experimental design and errors. We also use these results for correction of historical leveling conducted between Saugus and Palmdale along the Southern Pacific right-of-way since 1955. This 50-km-long grade (Figure 1) forms one important basis for the southern California uplift identified by Castle *et al.* [1976, 1983a, 1984], and Mark *et al.* [1981]. The U.S. National Geodetic Survey (NGS; previously named the U.S. Coast and Geodetic Survey) leveled this route in 1955, 1961, 1964, 1965, and 1971.

Geodetic leveling is a hundred-year-old optical measure-

ment technique designed to determine the elevations of bench marks embedded in the ground. The requisite equipment consists of a precise level and a pair of graduated rods. The rods are held vertically at equal distances from the horizontal level. The height between two bench marks, schematically illustrated in Figure 2a, is obtained by summing the height differences between the rods during each setup of the level instrument. The change in height, or divergence, can be measured by releveling, as long as the procedure and conditions during the initial and final survey are identical.

Although atmospheric refraction in geodetic leveling has been investigated with renewed interest [Angus-Leppan, 1979, 1984; Remmer, 1980; Shaw and Smietana, 1983; Brunner, 1984; Webb, 1984], the fundamental principles were first derived and tested by Kükkamaki [1938]. Temperature variations cause changes in air density along the sight path that have two consequences for leveling: scintillation and refraction. When heated air rises unstably through the overlying cooler and denser air, convective turbulence causes random fluctuations in the line of sight known as scintillation; on a hot day it can be recognized by the shimmering of objects on the horizon. Near the ground, radiant energy from the sun is conducted to the surface materials and returned to the atmosphere as heat and evaporated moisture, or latent heat. This process gives rise to vertical temperature gradients and temperature instabilities. The horizontal line of sight is refracted by the vertical gradient during leveling on sloped terrain (Figure 2b). If the temperature gradient were linear, the foresight and backsight would be deflected equally, and refraction

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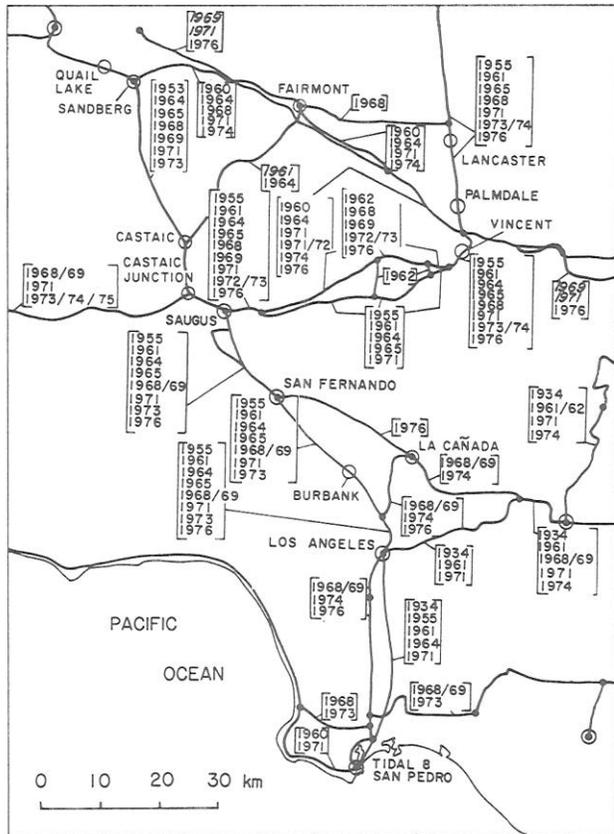


Fig. 1. Southern California leveling network in the vicinity of the Saugus-Palmdale grade [from Castle et al., 1984].

would cancel with each setup. A nonlinear temperature gradient causes a refraction inequality: The sight path is deflected most near the ground, where radiant heating is most pronounced, and so the uphill sight is refracted more than the downhill sight. If this refraction inequality is not maintained during every resurvey of a route, a systematic error will be introduced into the calculation of elevation change. To avoid this error, refraction must be modeled and removed.

The historical remedy for scintillation, shortening the sight length, has affected the accumulation of unequal refraction. As sighting accuracy was enhanced in the mid-1960s by introduction of the parallel plate micrometer and increased telescopic power of the level instrument, and by use of rods with two side-by-side scales, sight lengths observed on gentle topographic gradients in the United States were reduced. Sight lengths on the 1.3% grade between Saugus and Palmdale are limited only by atmospheric sighting conditions and survey tolerances; they diminished from a 60- to a 26-m average from 1955 to 1971 (Table 1). Because the line of sight through a medium of monotonically varying density describes a parabola, refraction has a squared dependence on sight length. Thus the refraction correction appropriate for a survey conducted before limitations were imposed on sight length could be as much as 4 times larger than the correction applied to a modern survey. There has been little shortening of sights on steep terrain, such as along the route from Saugus to Sandberg (Figure 1) because on grades greater than 4% (height/length = 0.04), the usable 2.5-m rod height (0.5–3.0 m above the ground) limits sights to less than 30 m.

EXPERIMENTAL PROCEDURE

We sought a simple way to conduct leveling as it was performed during 1955, when the sight length averaged 60 m, and as it is currently performed, with 25- to 30-m sights. The experimental leveling was double run, in which one leg of each section was composed of long-sight-length observations and the other was made with sights of approximately half the length. No attempt was made to secure a standard height free of refraction error. Instead, the differences in heights obtained with the long and short sights were examined. Those parameters thought to influence the accumulation of refraction were measured, while we attempted to minimize or randomize all sources of error unrelated to refraction. The temperature gradient, wind speed, cloud cover, precipitation, and ground cover were recorded at every instrument setup. The field crew leveled the route, which rises 612 m (Figures 3 and 4), for 23 days during the period May 14 to June 16, 1981. The same level instrument and rods were used for short- and long-sight leveling. The rods were calibrated at every graduation (5-mm intervals) by the U.S. National Bureau of Standards before the test. All leveling observations were corrected for level collimation (the level plumb), rod scale error (rod length), rod thermal expansion, and astronomic error (solid earth tides). We list the survey tolerances in Table 2 and the field equipment in Table 3.

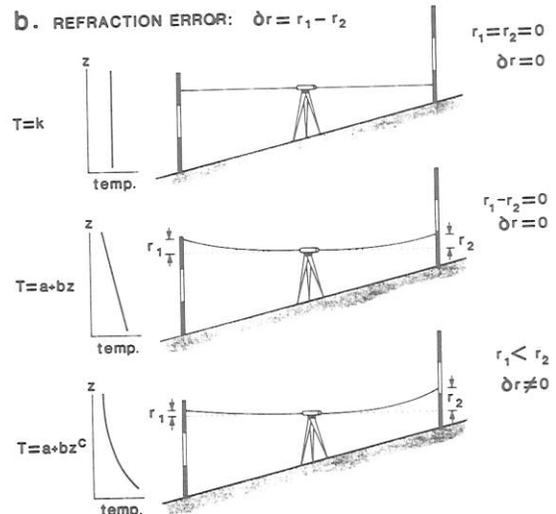
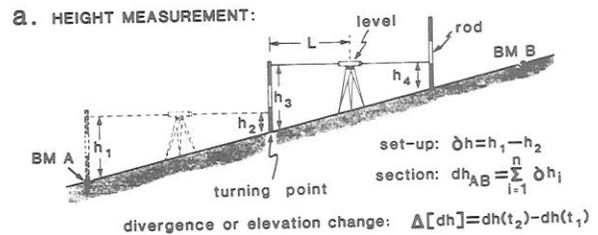


Fig. 2. (a) Leveling procedure, illustrating summation of height differences between rods,  $\delta h$ , to measure height difference between bench marks,  $dh$ , at time,  $t_i$ . (b) Dependence of refraction error,  $\delta r$ , on nonlinear vertical temperature gradient,  $T(z)$ .

TABLE 1. Elevation Change of BM N899 (7 km South of Palmdale) With Respect to BM X898 (at Saugus)

	Year of Survey					
	1955	1961	1964	1965	1971	1981
Survey period	May 2-16	May 18-31	April 23 to May 12	March 4 to May 19	May 19 to June 3	May 14 to June 16
Mean sight length, m	60	51	42	28	26	$S = 22, L = 46$
Refraction correction, mm	113	86	44	23	20	$S = 27, L = 79$
Field $dh$ , mm	0	38	196	142	96	99
Rod-corrected $dh$ ,* mm	0	31	103	143	135	114
Rod- and refraction-corrected $dh \pm \sigma$ ,† mm	0	$4 \pm 16$	$34 \pm 16$	$54 \pm 16$	$43 \pm 16$	$53 \pm 16$

\*Rod scale and index error, level collimation, astronomic (tidal), and orthometric corrections applied.  
 †Uncertainties: random error,  $\pm 7$  mm; rod error,  $\pm 7$  mm; refraction error,  $\pm 13$  mm.

TEMPERATURE RESULTS

Because leveling refraction is a consequence of the temperature structure of the lowest 3 m of the atmosphere, we measured the thermal gradient during all 1650 setups. Consistent nonlinearity of the vertical temperature gradient indicated that unequal refraction error does not cancel during leveling, as shown schematically in Figure 2b. Observations were well

fitted by an exponential temperature model

$$T = a + bz^c \tag{1}$$

where  $T$  is temperature,  $z$  is height in meters, and  $a$ ,  $b$ , and  $c$  are constants. The temperature exponent  $c = 0.22 \pm 0.02$  and  $b = -3.67 \pm 0.06$  K;  $dT = 1.34 \pm 0.02$  K, where  $dT$  is the temperature difference between the bottom ( $z = 0.5$  m) and top ( $z = 2.5$  m) probes (population standard deviations cited).

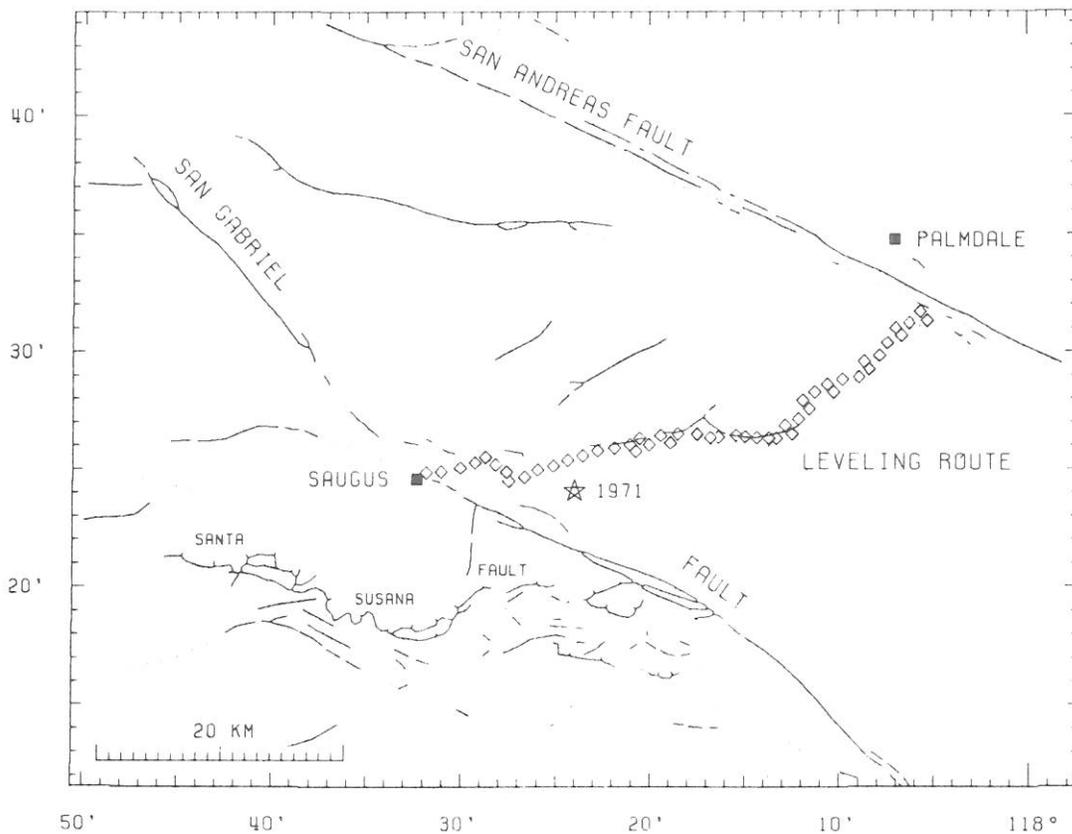


Fig. 3. Map of the Saugus-Palmdale leveling route amid active faults, showing epicenter of February 9, 1971, San Fernando, California,  $M_L = 6.4$  earthquake.

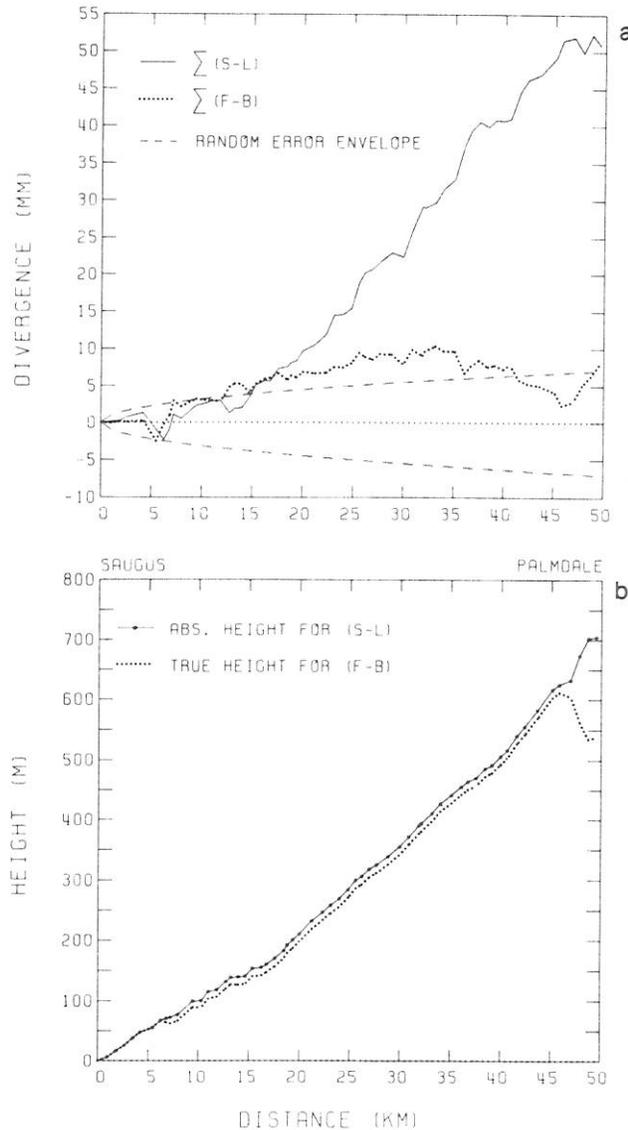


Fig. 4. (a) Observed divergence between short- and long-sight leveling  $\sum(S-L)$ , and divergence between sections leveled forward (toward Palmdale) and backward  $\sum(F-B)$  after correction for refraction error. (b) Topographic profile of the Saugus-Palmdale grade. True topography is used for  $\sum(F-B)$  to preserve direction, whereas absolute topography is used for  $\sum(S-L)$  to accumulate maximum effective height difference.

The very large standard deviation of a single observation of  $c$  ( $0.22 \pm 0.98$ ) reflects both the turbulence of the boundary layer and the dependence of  $c$  on local environmental conditions. The temperature difference was also found to be a linear function of  $T$ , in which

$$dT = -(0.33 \pm 0.06) + (0.069 \pm 0.002)T_0 \quad (2)$$

for 1610 observations, where  $T_0$  is  $T$  measured at 1.5 m above the ground (Figure 5). The regression coefficient for (2) is 0.58, and  $F = 823$ , significant at the 99.9% confidence level.

**Cloud cover.** Clouds screen the earth's surface from solar radiation. The NGS field crews have recorded a sun code, where 2 = full sun (less than 25% cloud cover), 1 = partial sun (25–75% cloud cover), and 0 = overcast (more than 75% cloud cover). Because these codes can be used during correc-

tion of historical observations for refraction, the dependence of  $dT$  and  $c$  on the indexes was tested.  $dT$  was reduced by  $30 \pm 2\%$  under partial sun, and by  $58 \pm 2\%$  during overcast conditions (Table 4).

**Wind.** Under the unstable thermal conditions that characterize daytime leveling, heated air near the ground rises through the overlying cooler air unless the wind is sufficiently strong to remove excess heat by forced convection at the thermal boundary layer. NGS leveling crews have traditionally recorded a three-index wind code, where 0 = calm conditions (0–2.8 m/s (0–10 km/h)), 1 = moderate winds, (2.8–7 m/s (10–25 km/h)), and 2 = strong winds (greater than 7 m/s). The peak wind velocity measured at 1.5 m above the ground during our experiment was 8 m/s. We found no difference in either  $dT$  or  $c$  significant at the 99% confidence level between wind codes 0 and 1 (Table 4).

**Ground cover.** More than half of the leveling observations were made on the gravel bed beside the railroad tracks. This well-drained material is dark and can be oil soaked where it underlies the rails. A few setups were made on black asphalt, light-colored concrete highway sidewalks, curbs, bridges, and soil, in some places with a sparse cover of vegetation. These surfaces, however, were not randomly distributed along the route. The most striking result of observations made on these six surfaces is the dependence of  $dT$  and  $c$  on the ground beneath the line of sight (Figure 6a).  $dT$  ranges from  $0.75 \pm 0.05$  K (concrete) to  $1.85 \pm 0.12$  K (asphalt) under cloudless conditions. The darker oil- and tar-soaked surfaces absorb and reradiate heat more efficiently than the more reflective, higher albedo surfaces. Because highway leveling is performed on both of these surfaces, the distinction is important for correction of historical leveling observations: A threefold to eightfold increase in refraction error was associated with leveling on asphalt in lieu of concrete (Table 5). The oil coating on the gravel beneath the rails and the tar matrix in asphalt also raise the heat capacity of these materials. The desert vegetation absorbs water and returns incident heat to resist desiccation, whereas water percolates through the gravel. Because no heat is consumed in evapotranspiration of surface water over these materials, the thermal gradient is large.

#### REFRACTION MODELS

We correct the long- and short-sight surveys of each section for refraction error and compare the refraction-corrected divergence (the difference in height measured with long and short sights) to the observed divergence, under the assumption that the long- and short-sight leveling should not diverge after correction for refraction error.

**Refraction formulas.** Kikkamaki [1938] developed corrections for leveling observations under the dual assumptions that isothermal surfaces within the lowest 3 m of the atmosphere align parallel to the ground surface and that the dependence of temperature on height within this thermal boundary layer can be represented by the power law (1), with  $c = -1/3$ . Angus-Leppan [1979, 1984] also constrained  $c = -1/3$ , under the free convection approximation [Webb, 1984]. Convection of the unstable boundary layer causes variations of the instantaneous temperature and its gradient; thus (1) can hold only for time-averaged and spatially averaged observations of temperature. For a single sight from the level instrument to the rod, the refraction correction to the observed elevation differ-

TABLE 2. Survey Tolerances

Attributes	Leveling		
	1981 Short Sight	1981 Long Sight	1955–1965 Epoch
Maximum difference in length of (forward-backward) sights			
Per setup, m	2	2	10
Per section, m	4	4	...
(Low-high) scale-elevation difference for steep $\delta$ , mm	0.30	0.75	...
Maximum section misclosure, mm	4.00	4.00	4.00

ence is given by

$$r_i = -d \cot^2 \varepsilon_i \frac{dT}{Z_2^c - Z_1^c} \left( \frac{1}{c+1} Z_i^{c+1} - Z_0^c Z_i + \frac{c}{c+1} Z_0^{c+1} \right) \quad (3)$$

where  $Z_i$  is the foresight or backsight,  $Z_0$  is the instrument height,  $\varepsilon$  is the ground slope between instrument and rod, and

$$\cot \varepsilon_i = L_i / (Z_0 - Z_i) \quad (4)$$

where  $L_i$  is the length of the sight. The air-density term is

$$d = [0.933 - 0.0064(T_0 - 293)](1 - bH/T_s)^{g/Qb} \quad (5)$$

where  $T_0$  is the air temperature at the instrument in kelvins,  $H$  is the height above sea level,  $g$  is the gravitational acceleration,  $Q$  is the gas constant, and  $b$  is the adiabatic lapse rate. The temperature at sea level,  $T_s$ , can be approximate by letting  $T_s = T_0 + bH$ . When the ground slope is nearly constant over a section and the foresight and backsights are balanced, as in the Saugus-Palmdale leveling, the refraction correction for a section simplifies to

$$r = -d \cot^2 \varepsilon \frac{dT}{Z_2^c - Z_1^c} \left[ \frac{1}{c+1} (Z_f^{c+1} - Z_b^{c+1}) - Z_0^c (Z_f - Z_b) \right] n \quad (6)$$

Here,  $\varepsilon \approx \tan \varepsilon = dh/S$ , and  $Z_f$  and  $Z_b$  are the average rod readings constructed from  $Z_f = Z_0 - \varepsilon L$  and  $Z_b = Z_0 + \varepsilon L$ , where the mean sight length per section  $L = S/2n$ ,  $S$  is the section length, and  $n$  is the number of setups in the section.

Use of (6) also enables averaging of 10–20 measurements of  $dT$  taken during leveling of a typical 1.5-km-long section.

Remmer [1980] argued that the inherent imprecision of field measurements of  $dT$  and  $c$  in a turbulent boundary layer means that a least squares adjustment of the data for sight length

$$r = -\eta L^2 dh \quad (7)$$

should remove refraction error as efficiently as the Kükkamaki expression. Here,  $\eta$  minimizes either leveling-circuit misclosures or the divergence between multiple surveys of a particular leveling route. Kükkamaki [1938] suggested an interpolated form of (6) for approximate correction of refraction error, in which the refraction error is assumed to vary linearly with height for sights made above 0.5 m off the ground

$$r = -D\gamma dT L^2 dh \quad (8)$$

where

$$\gamma = \frac{5.95}{Z_2^c - Z_1^c} \left\{ \frac{1}{c+1} [(Z_0 - 100)^{c+1} - (Z_0 + 100)^{c+1}] - (Z_1 - Z_2)Z_0^c \right\} \quad (9)$$

$D$  is the mean value of  $d$  for the leveling,  $r$ ,  $L$ , and  $dh$  are in meters,  $dT$  is in kelvins, and  $Z_1$ ,  $Z_2$ , and  $Z_0$  are in centimeters. Thus  $\eta$  in (7) can be interpreted as the product  $D\gamma dT$  in (8).

We tested for refraction error by using (6), (7), and (8). To measure the goodness of fit of the correction relative to uncertainty caused by random errors, we weighted the cumulative divergence, under the assumption that the random error  $\sigma = \alpha(S)^{1/2}$ , where  $\sigma$  is in millimeters,  $S$  is distance in kilometers,

TABLE 3. Field Equipment

Item	Description	Serial Number
Instrument	Jena NI002 automatic compensating level with optical micrometer.	45661
Rods	Kern, 1/2-cm graduation, unmatched. Calibrated before use by U.S. National Bureau of Standards by laser interferometry at every graduation, to an accuracy of $\pm 0.03$ mm or $\pm 10$ ppm at 99% level of confidence.	270718 277920
Temperature probes	Doric model 430A digital T-meter linked to three aspirated thermister probes; see Whalen [1981]. Calibrated by NGS Instruments and Equipment Branch on September 25, 1981.	81620
Anemometer	Air guide, hand-held; does not register below 1.4 m/s.	...

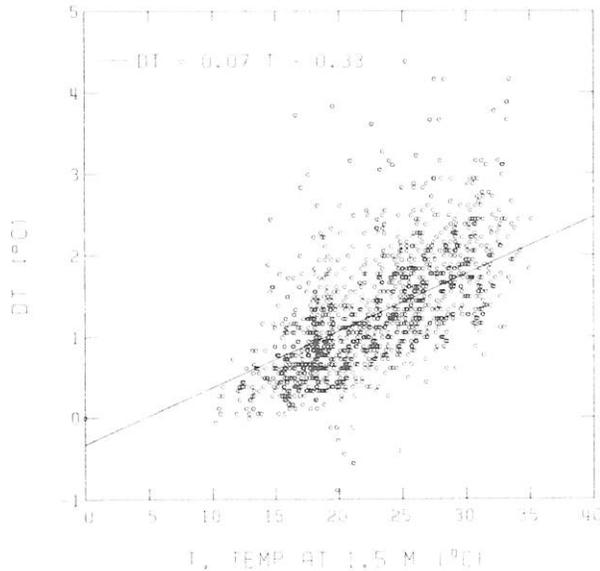


Fig. 5. Vertical temperature difference,  $dT$  ( $T_{0.5m} - T_{2.5m}$ ) as a function of temperature,  $T$  ( $T_{1.5m}$ ), showing a least squares regression line through the 1610 observations.

and  $\alpha$  is in millimeters per (kilometer) $^{1/2}$ . The rms error of the divergence is given by

$$(O - C) = \left\{ \sum [\text{div}(S - L)_i \omega_i]^2 / \sum \omega_i^2 \right\}^{1/2} \quad i = 1, n \quad (10)$$

where  $\text{div}(S - L)$ , the divergence between short- and long-sight leveling, is cumulative and  $\omega_i = 1/(\sigma_i)^2$ .  $(O - C)$  is thus in units of  $\sigma$ ;  $(O - C) \gg 1$  implies incomplete removal of error, and  $(O - C) = 1$  indicates that only random errors remain. For first-order class II leveling from the North American vertical datum,  $\alpha = 0.7$  mm [Federal Geodetic Control Committee, 1984]. Thus the uncertainty  $\sigma$  of the difference between two single-run surveys (the long- and short-sight surveys) is  $2^{1/2} (0.7 \text{ mm}) S^{1/2} = 1.0 \text{ mm } S^{1/2}$ . For the observed 51-mm  $\sum (S - L)$  divergence, the cumulative short-sight minus long-sight divergence  $(O - C) = 6.3$ . Therefore, without correction, the divergence is 6 times larger than expected from random error accumulation.

*Correction of the experimental data.* Adjustment of  $\sum (S - L)$  as a function of  $L^2$ , using (7), removes 75% of the divergence. Here,  $(O - C) = 1.63$  indicates that the residuals marginally exceed the estimate of expected random error (Figure 7, dotted lines). When the values of  $\gamma$  and  $dT$  averaged per section are used in (8), the removal of error improves:  $(O - C) = 0.94$ . Thus, use of temperature observations affords greater removal of the refraction error.

The observed  $\sum (S - L)$  divergence corrected by using the balanced-sight equation (6) with  $c = -1/3$  is shown in Figure 7 (solid line);  $(O - C) = 1.01$ . The fit is indistinguishable from the result from the simpler (7). Unusually small variations ( $\pm 8\%$ ) of  $d$  in (5) were noted during the experiment, however, because the temperature increased rather than decreased with elevation. This increase occurred because spring advanced as the crew leveled toward the summit of the grade. When the observed  $c$  per section was used, (6) undercorrected the error by 17%, and  $(O - C) = 1.4$ . This suggests that measurement of the second derivative of the vertical temperature gradient with three temperature probes is inadequate; letting  $c = -1/3$  is sufficient to remove refraction error.

*Correction without measurement of the temperature gradient.* Historical leveling data lack measurements of  $dT$  and  $c$ , although a measurement of  $T$  at 1.5 m was made at the start and end of each section to correct for thermal expansion of the rods. Holdahl [1981] developed a temperature-stratification model based on historical records of solar radiation, sky cover, precipitation, and ground albedo

$$dT = T_1 - T_0 = 3 \left[ \frac{H^2 T_0}{(C_p \rho)^2 g} \right]^{1/3} (Z_1^{-1/3} - Z_0^{-1/3}) \quad (11)$$

where  $H$  is the upward sensible heat flux,  $C_p$  is the specific heat of the air at constant pressure, and  $\rho$  is the air density. Implicit in (11) is the assumption that  $c = -1/3$ . When values of  $dT$  predicted from (11) replaced the observed  $dT$  in the balanced-sight equation (6),  $(O - C) = 1.9$  (see Figure 8b), similar to the fit for (7). Although the mean  $dT$  predicted by (11) for the experiment differs by only 0.11 K from that observed (predicted  $dT = 1.29 \pm 0.027$  K, observed  $dT = 1.38 \pm 0.053$  K), the predicted values depart systematically from those observed along the grade (Figure 6b). The linear relation between  $T$  and  $dT$  deduced for the experiment (Figure 5) suggests that (2) may be used in place of (11) for Saugus-Palmdale leveling conducted during late spring-early summer. When the balanced-sight equation (6) is used with  $c = -1/3$  and the  $dT$  predicted from (2), the removal of refraction error is superior to that with  $dT$  predicted from (11):  $(O - C) = 1.03$  (Figure 8b, dotted line).

Because the temperature difference varied as a function of the ground surface, an alternative method to generate  $dT$  values is suggested: The mean  $dT$  predicted from (11) for the experiment is modified as a function of ground surface with the ground  $dT$  factors derived from Table 5. Refraction correction with ground-based  $dT$ , using (6), is shown in Figure 8a (dotted line):  $(O - C) = 0.88$ . Because the 1955 and 1961 leveling field books do not specify whether the surveys were conducted on the railroad tracks or on the access road, application of this method to the historical leveling data is uncertain.

## DISCUSSION

Use of the balanced-sight equation to eliminate atmospheric refraction error from the experimental Saugus-Palmdale leveling has proved both practical and accurate. Field experiments conducted elsewhere merit comparison to the southern California test. They reinforce the result that Kükkamäki's [1938] single- or balanced-sight equations (3) and (7) provide suitable tools for the removal of refraction error.

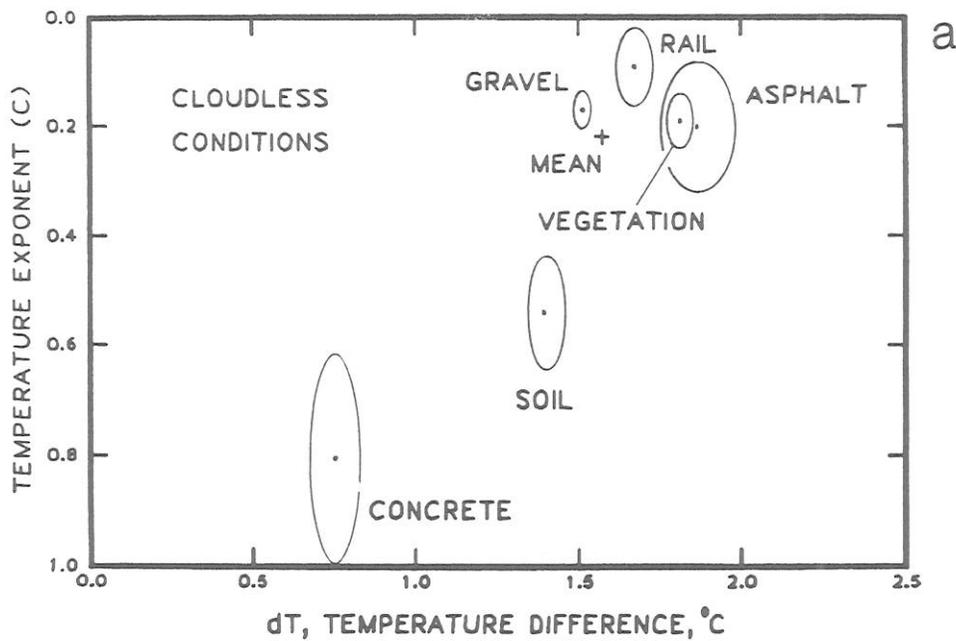
*Comparison with other field tests.* Heroux et al. [1985] conducted a leveling field test in Canada similar to the Saugus-

TABLE 4. Temperature Dependence on Cloud Cover and Wind Speed

Condition	NGS Code	Observed $dT$	Percent of Maximum $dT$	Observed Exponent $c$	$n$
Full sun	2	$1.50 \pm 0.03$	100	$0.16 \pm 0.03$	654
Partial sun	1	$1.06 \pm 0.05$	$70 \pm 2$	$0.16 \pm 0.09$	174
Overcast	0	$0.63 \pm 0.04$	$42 \pm 2$	$0.44 \pm 0.13$	140
Wind 2.8–7 m/s	1	$1.58 \pm 0.04$	100	$0.16 \pm 0.05$	271
Wind < 2.8 m/s	0	$1.44 \pm 0.03$	$91 \pm 5$	$0.16 \pm 0.05$	383

All observations on gravel ground surface.

(C) AS A FUNCTION OF  $dT$  FOR 6 GROUND SURFACES



OBSERVED - PREDICTED TEMPERATURE DIFFERENCE PER SECTION

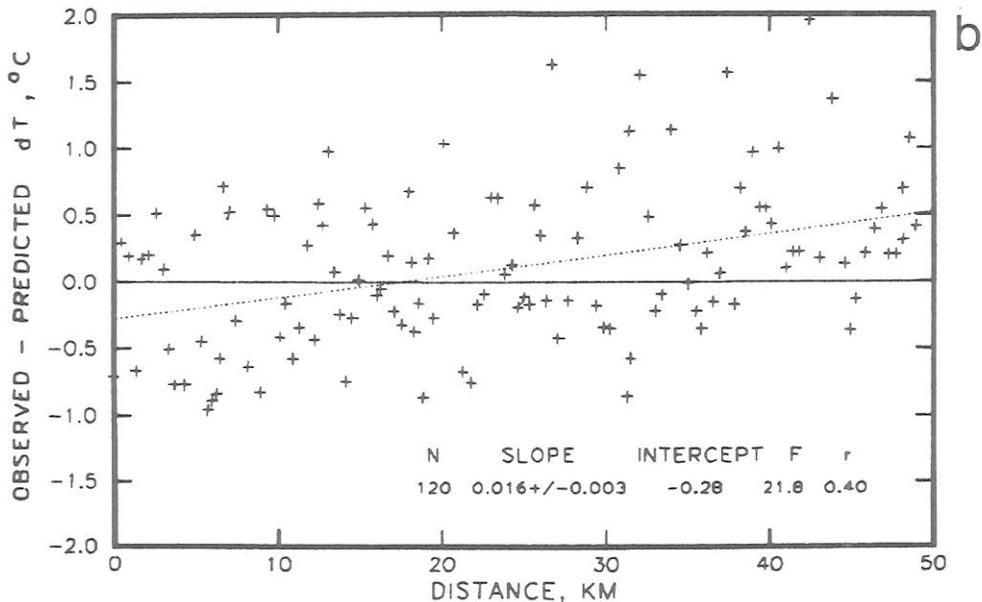


Fig. 6. (a) Observed temperature exponent  $c$  as a function of observed temperature difference,  $dT$ , for six ground surfaces (population standard deviations shown). (b) Observed  $dT$  minus predicted  $dT$  from (11), showing a statistically significant trend in the difference (dotted).

Palmdale experiment, in which the accumulated refraction error was successfully modeled by the single-sight equation (3). The lengths of the long sights (mean, 45 m) and short sights (mean, 22.5 m) were similar to the sight lengths in the Saugus-Palmdale experiment. The same model of level instrument was used in both the U.S. and Canadian experiments. Fourfold releveling of a 6-km-long route in Quebec resulted in an ef-

fective height gain of 293 m over 24 km, 40% of the height gain between Saugus and Palmdale. The mean observed  $dT$  was also 40% of that measured in southern California,  $0.55K$ . The  $\sum (S - L)$  divergence was  $14 \pm 7$  mm; after correction, it was  $0 \pm 7$  mm.

Whalen [1981] conducted a fixed refraction experiment in which 20- to 60-m sights were made at sites in Maryland

TABLE 5. Summary of Temperature Observations Under Cloudless Conditions

	All Data	Concrete	Soil	Gravel	Vegetation	Asphalt	Rail
Number of observations $n$	1136	56	108	654	178	33	128
$dT$ [ $T(z_1) - T(z_2)$ ], K	$1.54 \pm 0.02$	$0.74 \pm 0.05$	$1.38 \pm 0.06$	$1.50 \pm 0.03$	$1.80 \pm 0.05$	$1.85 \pm 0.12$	$1.66 \pm 0.06$
Ground $dT$ factor	1.0	0.5	0.9	1.0	1.2	1.2	1.1
Exponent $c$ of equation (1)	$0.22 \pm 0.03$	$0.81 \pm 0.21$	$0.53 \pm 0.10$	$0.16 \pm 0.03$	$0.18 \pm 0.06$	$0.19 \pm 0.12$	$0.08 \pm 0.07$
$\gamma$ in equation (8)	$46 \pm 2.4$	$12 \pm 13$	$29 \pm 6$	$48 \pm 2$	$47 \pm 3$	$46 \pm 6$	$52 \pm 3$
Refractivity $\gamma dT$	$71 \pm 4$	$10 \pm 11$	$40 \pm 8$	$72 \pm 3$	$85 \pm 6$	$85 \pm 12$	$86 \pm 6$
Refractivity/mean refractivity	$1.0 \pm 0.08$	$0.14 \pm 0.11$	$0.56 \pm 0.21$	$1.01 \pm 0.07$	$1.20 \pm 0.09$	$1.20 \pm 0.15$	$1.21 \pm 0.09$

(mean  $dT = 0.56$  K) and in Arizona (mean  $dT = 1.03$  K). Whalen removed about 95% of the observed cumulative divergence with Kükkamaki's single-sight equation (3), using the observed  $dT$ , comparable to the results obtained along the Saugus–Palmdale grade. Correction of the divergence with the  $dT$  predicted from (11) was generally more successful in the Maryland and Arizona experiments than in the southern California field test. *Banger* [1982] also measured the temperature gradient and leveling refraction errors with a fixed field test in Turkey, removing 85–95% of the errors obtained with sights lengths of 30–50 m by using the single-sight equation (3).

*Independent analysis of the Saugus–Palmdale experiment.* *Shaw and Smietana* [1983] developed a model of temperature stratification from Monin–Obukhov similarity theory, which accounts for mechanically induced mixing of the boundary layer caused by wind. They found that the free convection approximation used in Kükkamaki's formulas and in Holdahl's predicted  $dT$  gives an upper bound on refraction error. The free convective approximation yielded estimates of refraction error 10–50% larger than did the similarity theory. *Shaw and Smietana* [1983] also modeled the leveling from the Saugus–Palmdale experiment. They estimated friction velocity and surface heat flux from the observed temperature gradient and wind velocity. Exact corrections could not be made because the roughness length of the ground surface was not measured and because the wind velocity measurements proved inadequate for the purposes of their model. *Shaw and Smietana* [1983], instead, presented a suite of corrections for the 42-mm cumulative divergence measured over the true topography (Figure 4b). The Monin–Obukhov theory gave 49 mm for a roughness length of 0.4 m, whereas the free convection representation of refraction error of *Angus-Leppan* [1979] gave 70 mm. It is difficult to assess the effectiveness of *Shaw and Smietana's* correction without complete measurements of roughness length and wind speed. A longer mean roughness length appears necessary to correct the Saugus–Palmdale observations as well as was accomplished by (6).

#### EXPERIMENTAL ERRORS

*Castle et al.* [1983a, b, 1984] have argued that past standards and practices precluded significant refraction errors in the historical leveling from 1955 to 1965. *Castle et al.* [1983b] and *Craymer and Vaniček* [this issue] also maintain that systematic errors unrelated to atmospheric refraction contaminated the 1981 experimental results. We now address these important issues.

*Leveling standards and specifications.* The long-sight leveling conducted during the experiment meets all the standards and specifications for first-order leveling performed during 1955–1965, which it was designed to duplicate; the short-sight leveling satisfies all the standards and specifications currently

in force for first-order class II leveling. During 1955–1965, first-order leveling field specifications stipulated that the height difference measured between bench marks during the two runnings of each section (in the backward and forward direction) must be less than  $4.0 \text{ mm } (S)^{1/2}$  95% of the time, where  $S$  is the section length in kilometers. Since 1971, the Saugus–Palmdale leveling was run to the more stringent  $3.0 \text{ mm } (S)^{1/2}$  specification. In the 1981 experiment, 1 out of the 60 sections exceeded the  $4.0 \text{ mm } (S)^{1/2}$  criteria and was rerun (Figure 9a); the mean closure of all sections was  $+0.84$  mm. Thus the leveling met the 1955–1965 specifications. After correction for refraction, all sections closed within  $3.0 \text{ mm } (S)^{1/2}$ , and the mean misclosure reduced to  $+0.07$  mm (Figure 9b). Once corrected for refraction, the leveling thus satisfies the current specifications for section misclosure. The leveling also meets current first-order standards for the random error per section: For both single- and double-run first-order leveling currently conducted by the NGS, *Whalen and Balazs* [1976] found that the standard error per section

$$\hat{\sigma} = \frac{1}{2} \left\{ \sum [(F - B)_i^2 / S_i] / n \right\}^{1/2} \quad i = 1, n \quad (12)$$

is 0.7 mm, where  $n$  is the number of sections. For the refraction-corrected divergence in the Saugus–Palmdale experiment,  $\hat{\sigma} = 0.68$  mm.

The 1024 short-sight-length setups were conducted to the

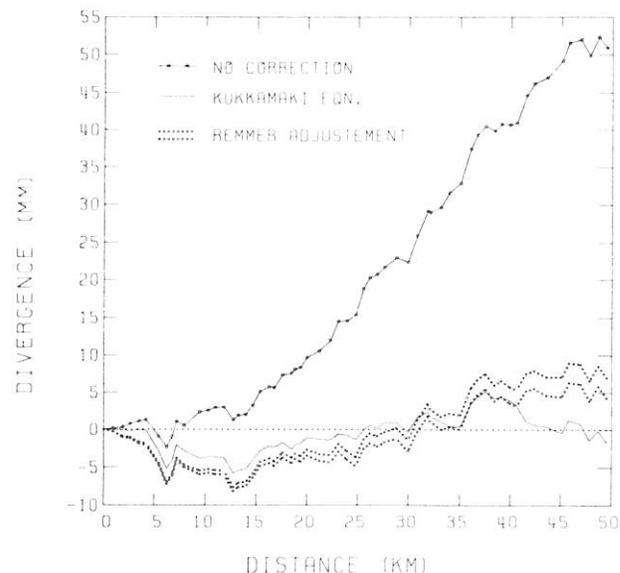


Fig. 7. Observed  $\sum (S - L)$  divergence, in comparison with  $\pm 1\sigma$  bounds on least squares adjustment for sight length ( $L^2$ ) suggested by *Remmer* [1980], and correction of refraction error by using balanced-sight equation (6) of *Kükkamaki* [1938].

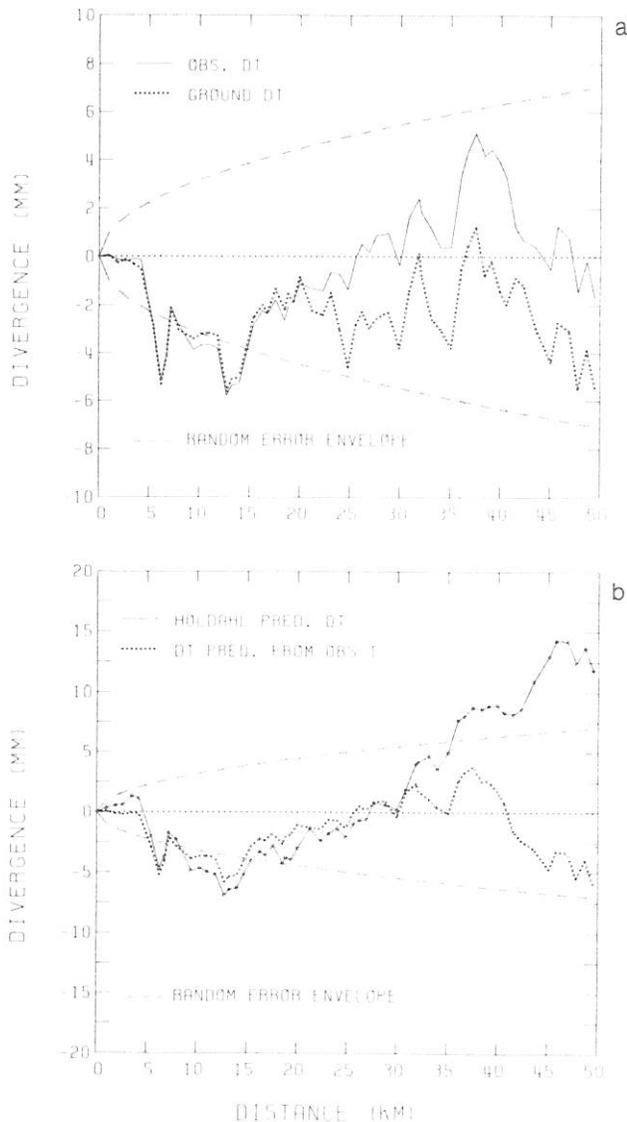


Fig. 8. Correction for refraction error under different assumptions. (a) Balanced-sight equation (6), with observed  $dT$  (solid line) and  $dT$  assigned by ground type (dotted), in comparison with  $\pm 1\sigma$  random error envelope assuming  $\sigma = 1.0 \text{ mm } (L)^{1/2}$ . (b) Refraction correction using approximations of temperature difference, with  $dT$  predicted according to (11) (solid), using (2) (dotted).

specifications given by Whalen and Balazs [1976] for first-order class III double-simultaneous leveling (now reclassified as first-order class II; see Federal Geodetic Control Committee [1984]). Forward and backward sights are made to two scales on each rod; the setup is rejected if the two measurements of height difference between the forward and backward rods (referred to as the “high scale” and “low scale” differences) do not agree within a prescribed tolerance  $\delta$ . The tolerance now in force,  $\delta \leq 0.30 \text{ mm}$  [Federal Geodetic Control Committee, 1984], was used for the short sights. The 590 long-sight-length setups were made by using the same procedures as for the short sights, except that the setup tolerance  $\delta$  was relaxed to 0.75 mm to attain an equivalent number of acceptable sights as for the short-sight observations (Table 2). The (high-low) scale differences for the sights are normally distributed about a near-zero mean, a result consistent with the absence of systematic collimation or settling error.  $Q - Q$  plots of observed

versus predicted quantiles of the (high-low) scale differences are shown in Figures 10a and 10b, following the method of Kleiner and Graedel [1980]. Along the  $x$  axis are plotted the quantiles predicted for a normal probability distribution; observations that fall on the dotted line are thus normally distributed. The distributions are short-tailed because observations that exceeded the rejection criteria,  $\delta$ , were not recorded.

The percentage of rejected sights was estimated under the assumption that the rejected sights obey the same normal distribution as the accepted sights. Using the population standard deviation that yields a  $Q - Q$  slope closest to 1.00, we find that 1% of the long sights exceeded  $\delta = 0.75 \text{ mm}$  and 3% of the short sights exceeded  $\delta = 0.30 \text{ mm}$ . The percentage of rejected sights is similar for both long- and short-sight surveys and is consistent with first-order standards of accuracy. The sighting statistics accord well with the sights made during the Canadian experiment, using the same sight lengths [Heroux et al., 1985]. The Canadian mean (high-low) scale height difference for short weights was  $-0.04 \pm 0.17 \text{ mm}$  and for long sights  $-0.04 \pm 0.29 \text{ mm}$ . The Saugus Palmdale experiment achieved nearly the same quality of sights under more refractive conditions than prevailed during the Quebec experiment.

Castle et al. [1983b] contended that the long sights made during the experiment did not meet the standards of accuracy practiced during 1955–1965, arguing that the poorest sights were associated with the largest refraction error. Both scintillation (random fluctuations in the line of sight) and refraction (systematic deflection of the line of sight) increase with sight length, and so long sights are inherently less precise than

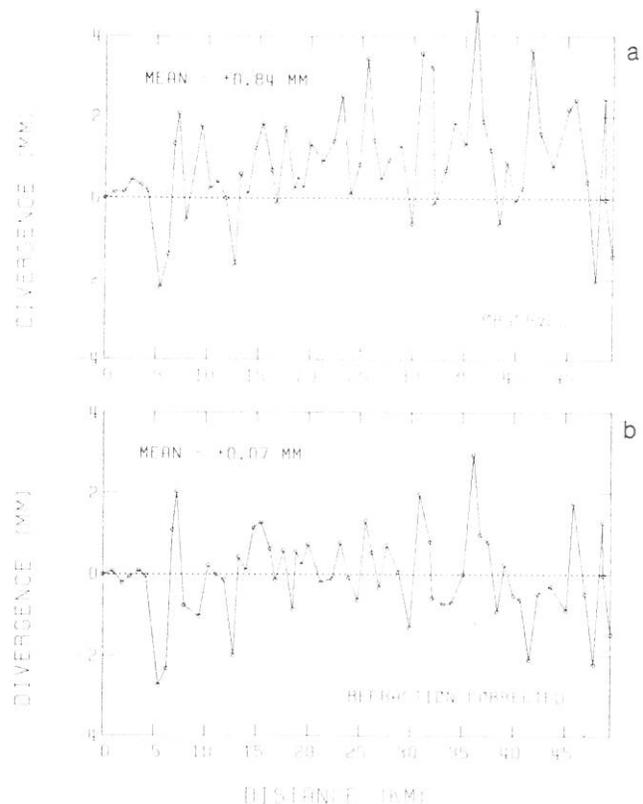


Fig. 9. Sectional divergence (not summed) of (a) observed and (b) refraction-corrected leveling. Before correction, one section exceeds the 1955–1965 4.0-mm  $(S)^{1/2}$  specification; after correction, all sections meet the more stringent 3.0-mm  $(S)^{1/2}$  specification for first-order geodetic control surveys currently in use.

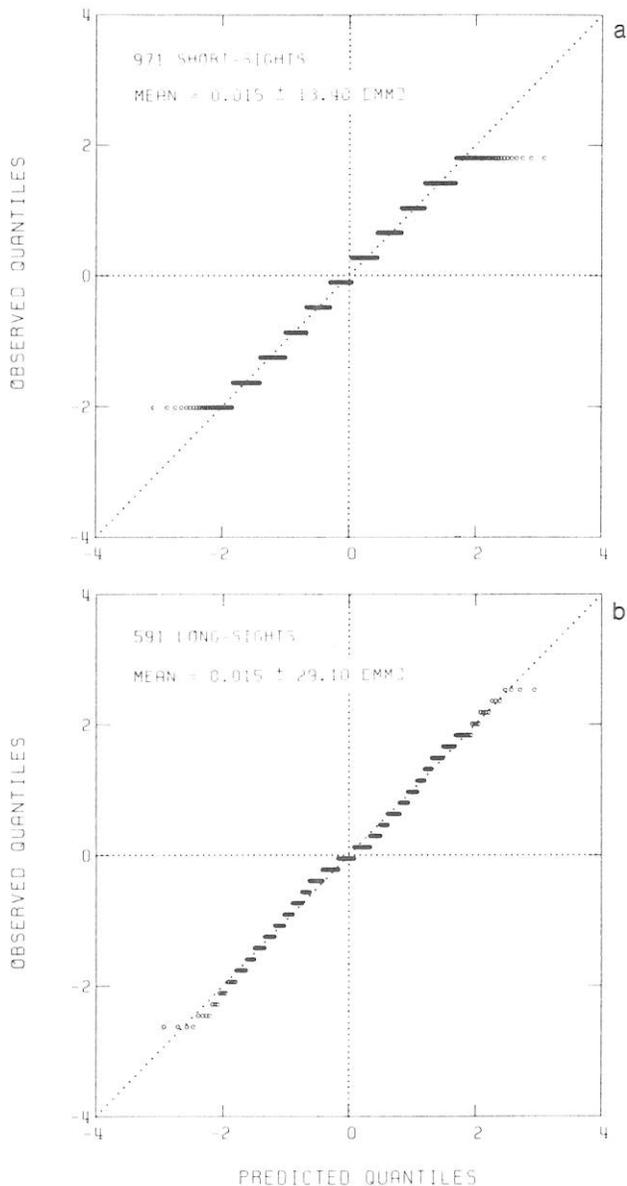


Fig. 10.  $Q - Q$  plots of (high-low) scale differences for (a) short-sight and (b) long-sight leveling. A normally distributed sample falls on the dotted line. Distribution is short-tailed because no observations were recorded outside of an acceptable range  $\delta$ .

shorter sights. The important question is not whether poor sights are more refractive but whether such sights were made during 1955–1965.

No rigorous setup rejection criteria existed before 1965. The 1948 field manual [Rappleye, 1948] instead instructs surveyors to choose sight lengths such that 85–95% of the sections meet the 4.0-mm ( $S$ )<sup>1/2</sup> section specification. The Fischer level used by the U.S. Coast and Geodetic Survey until 1961 was read by estimating the position of three horizontal reticles (wires) to the nearest millimeter on a single scale graduated at every centimeter on the rod. Under “length of sights,” the 1948 manual states [Rappleye, 19948, p. 7]

Observers have found that a convenient rule, in fixing the length of sight, is to shorten the sights whenever the upper and lower intervals subtended on the rod are found to differ frequently by more than a selected limit. Each observer

should fix this limit from his own experience, by noting the relation between a provisional limit and the amount of rerunning found to be necessary while using it. Such a rule is based on the idea that the additional errors which are encountered when the length of the sight is increased are, in the main, those due to the increased accidental errors in reading the rod.

It is now known that errors encountered with increased sight lengths are not due to accidental reading of rods but to refraction errors. Because the subtended intervals (the height difference between center and distal reticles) were only estimated to the nearest millimeter, the most restrictive standard possible was  $\delta = 1$  mm (i.e., rejecting all sights for which the thread intervals do not agree exactly). In 1955 the observer used a 3-mm tolerance (associated with a 60-m average sight length), and in 1961 a 2-mm tolerance was adhered to (with  $L = 51$

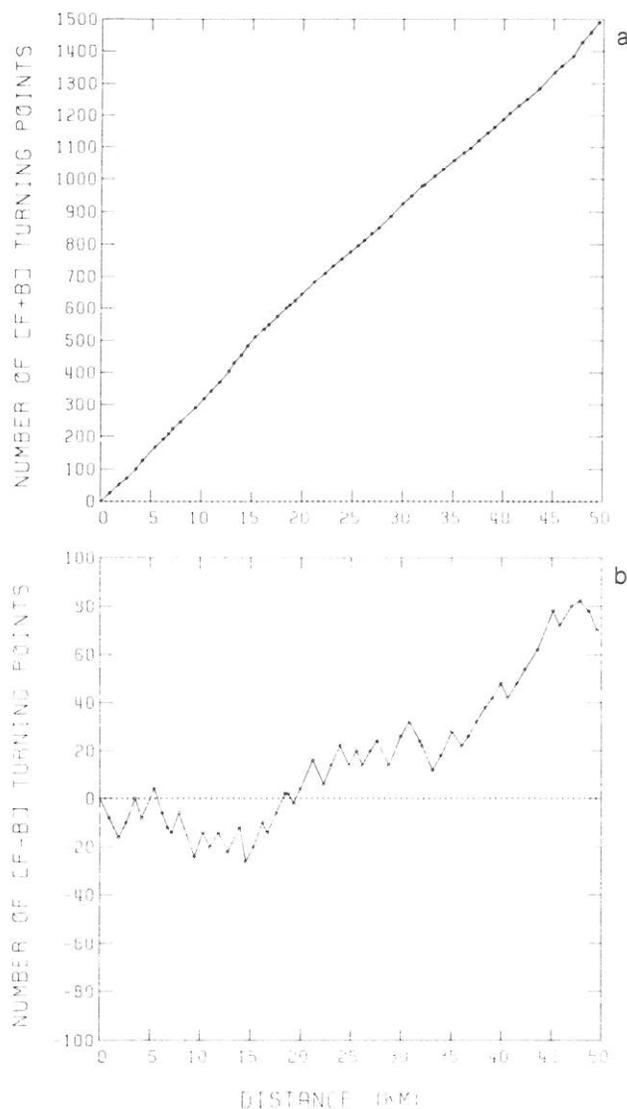


Fig. 11. Number of turning points as a function of distance. (a) Number of turning points used in forward and backward running of each section. (b) Number of turning points used in running forward minus that used running backward, equivalent to  $F - B$  setup imbalance. Proper balance is maintained until km 35, after which more setups are made in running forward. At km 46, cumulative imbalance is 5%. Setup imbalance exhibits no statistical correlation with ( $F - B$ ) divergence (see Figure 4a);  $r = 0.20$ .

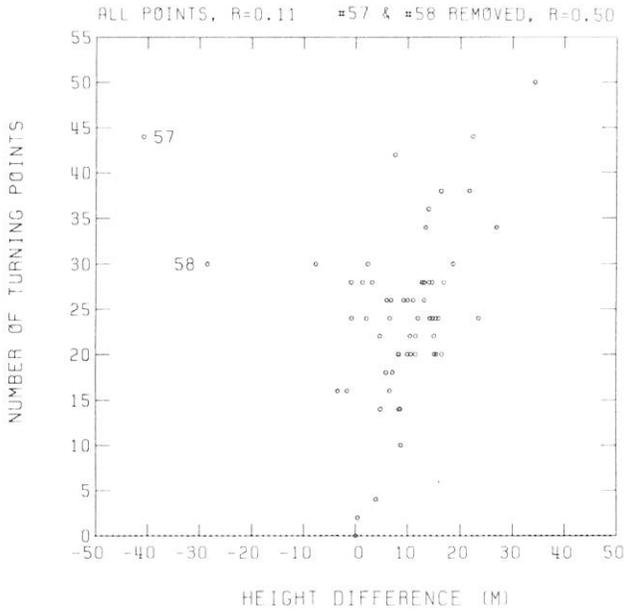


Fig. 12. The number of turning points per section as a function of the height difference per section. Removal of outliers 57 and 58 reveals that these functions are positively correlated at the 99.9% level of confidence.

m). Thus the sighting standards of the 1981 long-sight survey ( $\delta = 0.75$  mm) were tighter than those in 1955–1965 ( $\delta = 2-3$  mm).

No practice performed with a three-wire instrument and a single-scale rod graduated at centimeter intervals can be as restrictive as the sighting criteria established after 1965 for levels with optical micrometers and higher telescopic power sighting on double-scale rods. The historical three-wire test measured the quality of a single sight from instrument to rod, whereas the modern setup test measures the quality of the height difference carried from backsight to foresight. We chose not to level long sights with the Fischer level and rods used before 1965 in the 1981 experiment because use of different instruments and rods for long and short sights could have introduced additional sources of systematic errors into the experiment. Had a  $\delta = 0.30$  mm tolerance been used for both long and short sights in the experiment, about 28% of the long sights would have been rejected, a result inconsistent with the 1948 leveling guidelines that no more than 5–15% of the leveling should be rejected because it exceeds the sectional tolerance.

There is no evidence to suggest that historical leveling was curtailed, or that sight lengths were shortened, during periods of increased refraction. The 1955 and 1961 surveys commenced at 0745–0845, paused at 1200–1230 for 30 min, and stopped at 1600–1645 PST. The 1981 survey started at 0755–0815, paused at 1130–1230 for 60–90 min, and stopped at 1630 PST. In 1955, sight lengths during 0800–0930 were within  $\pm 5\%$  of those at 1230–1400 PST. During the 1981 experiment, the peak  $dT$  was recorded between 1200 and 1400 PST. Under “systematic errors on slopes,” the 1948 leveling manual encouraged crews to survey during the most refractive time of day, in the mistaken belief that the vertical temperature gradient is smallest near noon [Rappleye, 1948, p. 40].

The refraction error on a clear day should be at a minimum during the several hours in the middle of the day, when the

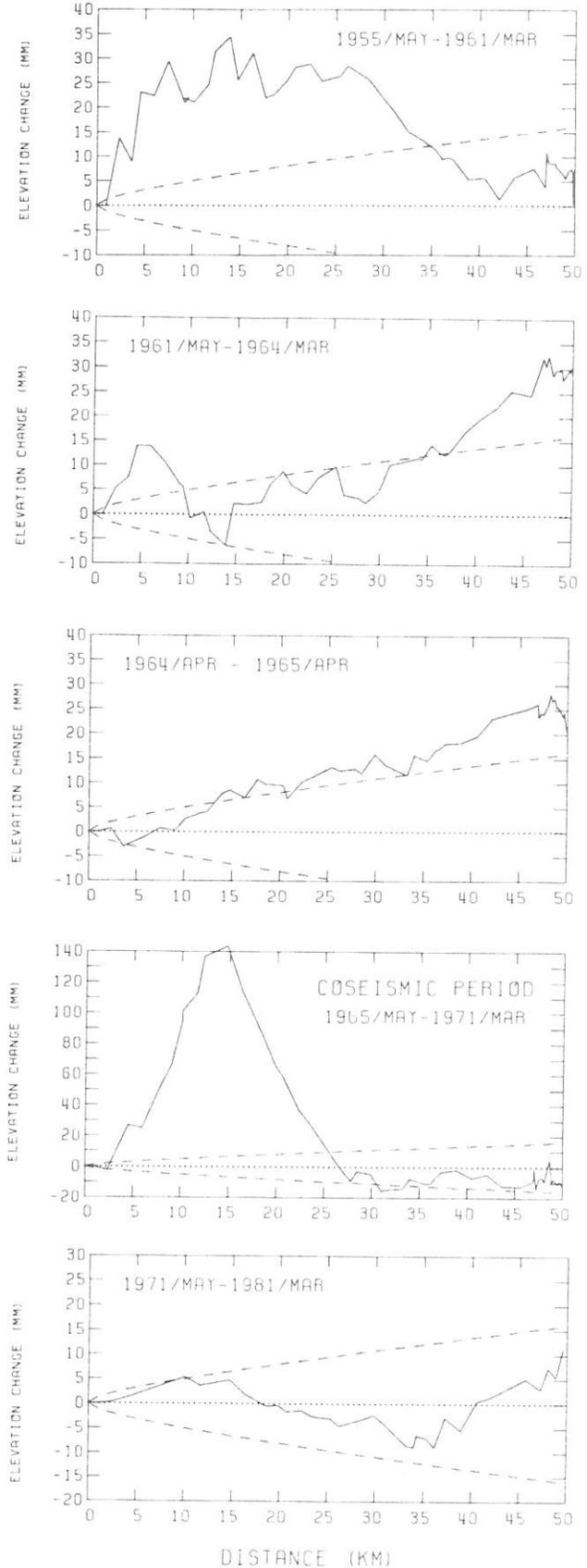


Fig. 13. Elevation changes for historical levelings of the Saugus-Palmdale grade relative to measurement uncertainty, after removal of rod-calibration error for 1964 and refraction error, using balanced-sight equation (6) and  $dT$  predicted from (11). Note that coseismic period is shown at one-third scale of other epochs.

temperatures of the air and the surface of the ground are nearly the same, and the greatest in the morning and afternoon, but should have opposite signs during these two periods.

In sum, the 1981 experiment was performed along the same route and during the same season and time of day as for the historical surveys; sights made during the experiment were no longer than those made historically, and all the specifications for first-order leveling in force at the time were satisfied. Thus the 1981 long-sight leveling is consistent with past specifications, standards, and practices.

*Directionally dependent error.* Castle *et al.* [1983b] suggested that a directionally dependent error which cannot be separated from refraction contaminated the Saugus–Palmdale leveling experiment. The observed cumulative divergence of all sections run forward (toward Palmdale) minus all sections run backward,  $\sum (F - B)$ , is 22.5 mm, 2.5 times larger than the expected random error. However, 35 out of the 60 long sections were run in the forward direction, biasing the  $\sum (F - B)$  to mimic the  $\sum (S - L)$  divergence. After refraction correction,  $\sum (F - B) = 8$  mm (Figure 4a, dotted line) and  $(O - C) = 1.8$ . Thus the refraction corrected  $\sum (F - B)$  is only marginally greater ( $< 2\sigma$ ) than the expected uncertainties.

Although the data do not exhibit correlations that typically reveal rod settlement error, progressive sinking of the rod turning pins during the setups may account for some of the 8-mm  $\sum (F - B)$  divergence. Settlement of the turning pins is identified by correlation of the  $(F - B)$  divergence with the number of turning points (the rod supports between bench marks, Figure 2a). Because of the constant grade of the Saugus–Palmdale route and the constant sight lengths maintained, the number of turning points is correlated with height (Figure 4b) and distance (Figure 11a). None of these three functions exhibits a correlation with the refraction-corrected  $(F - B)$  divergence, nor does the refraction-corrected  $(F - B)$  divergence correlate with the  $F - B$  setup imbalance (number of setups run forward minus those run backward, Figure 11b) or with the  $F - B$  time imbalance (time elapsed running forward minus that running backward).

The setup imbalance inadvertently deteriorates after the first 35 km of the grade and reaches a cumulative imbalance of 70 setups (5% of the total setups) at Palmdale (Figure 11b). This imbalance could result in a cumulative error of 7 mm if the sinking error were as large as 0.1 mm per setup. Bomford [1971, p. 238] reported sinking errors for turning pins of 0.010–0.024 mm per setup on road, rail, and sandy ground. Regardless, an error of 0.1 mm per setup would only slightly affect the experimental results because it would amount to only 35% of the observed  $\sum (S - L)$ .

*Multiple error sources.* Craymer and Vaniček [this issue] concluded that systematic errors accumulated with both rod turning points and height during the 1981 experiment. They determined that the +8 mm cumulative  $(F - B)$  divergence is composed of +20 mm of setup-correlated error (which they attribute to turning point settlement) and –14 mm of height-correlated error (which they ascribe to a 30-ppm rod scale error). Neither function, however, is correlated with the divergence in a simple linear regression. Thus in the multiple linear regression used by Craymer and Vaniček [this issue], the two functions must be independent. Height and turning points are by the design of the experiment, however, positively correlated. Because the slope of the leveling route is constant until the ridge crest at section 57 (see dotted line in Figure 4b),

longer sections have larger height gains. Because the sight lengths are constant, longer sections also require more setups or turning points. Thus height and the number of turning points both increase with section length.

The relationship between height and turning points is shown in Figure 12. The height difference per section appears uncorrelated with the number of turning points per section (Figure 12), when all 60 sections are included (the correlation coefficient,  $r = 0.11$ , with the number of degrees of freedom,  $n = 58$ ). When one section is removed (section 57), however, the correlation coefficient becomes statistically significant at the 99.5% confidence level ( $r = 0.36$ ,  $n = 57$ ). The slope of the regression is then 3 times larger than its standard deviation, the  $F$  statistic is significant at the 95% level ( $F = 8.72$ ,  $n = 57$ ), and the  $t$  test for the difference of the regression coefficients before and after the removal of the section is significant at the 94% level of confidence ( $t = 1.61$ ,  $n = 115$ ). When two outliers are removed (sections 57 and 58), the correlation is significant at 99.9% ( $r = 0.50$ ,  $n = 56$ ),  $F$  is significant at 99.95% ( $F = 18.7$ ,  $n = 56$ ), and  $t$  is significant at the 99.7% confidence level ( $t = 2.76$ ,  $n = 114$ ). Removal of just one or two sections at the end of the line (2–3% of the population) reveals that height and the number of turning points are not independent; thus the multiple linear regression of Craymer and Vaniček [this issue] is not robust. The fact that height and turning points are positively correlated also makes their conclusion that each function contributes errors of opposite sign untenable.

#### CORRECTION OF HISTORICAL SAUGUS–PALMDALE LEVELING

We used the results of the field test to correct the 1955–1981 leveling surveys for atmospheric refraction error with the balanced-sight equation (6) and the solar radiation model (11). The corrected elevation changes for each epoch are plotted in Figure 13, and the detailed data for the end points are listed in Table 1. After correction, aseismic uplift near Palmdale during 1955–1965 sums to  $65 \pm 16$  mm at km 47 (54 mm at km 50, 7 km south of Palmdale), a value that is significant at the 99% confidence level but is about one third the uplift inferred by Castle *et al.* [1984] without refraction correction. Pumping of water from the unconfined alluvial aquifer beneath Saugus is estimated to have caused  $9 \pm 3$  mm of local subsidence at Saugus [Stein, 1981, p. 453]. Thus the tectonic uplift of Palmdale with respect to Saugus is  $56 \pm 16$  mm during 1955–1965, significant at the 95% level of confidence. The cited 16-mm uncertainty in elevation change comprises random error (7 mm), rod scale error (7 mm), and residual refraction error (13 mm). The refraction residual reflects imperfect removal of refraction and, possibly, other errors from the experimental divergence when using (6) and (11). In the experimental calculation,  $(O - C) = 1.9$  (Figure 8b), and so we have assigned the uncertainty in the refraction correction to be  $1.9\sigma$ , where  $\sigma$  is the random error.

A significant correction was made to remove rod-scale error from the 1964 survey (NGS rods 312–318, 312–274). The unusually large ( $121 \pm 7$  ppm) scale error for these rods, which was discovered by Jackson *et al.* [1980], has since been investigated by Jackson *et al.* [1981], Strange [1980, 1981], and Stein [1981]. This error, which results in a correction of 70 mm over the 582-m-high grade, was found from a slope-dependent correlation of divergence where these rods were used on the adjacent Saugus–Sandberg route (Figure 1), and

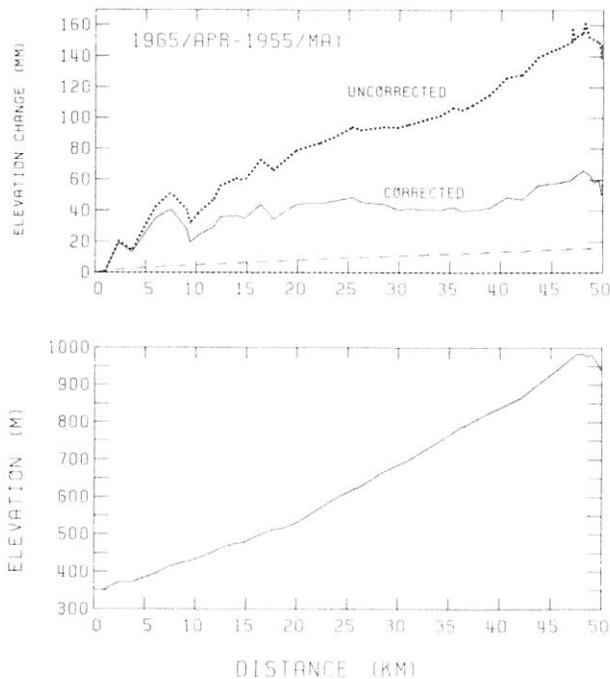


Fig. 14. Comparison of elevation changes with and without correction for refraction error.

from inconsistent rod calibration certificates (Castle *et al.* [1983a, p. 2510], however, disputed this finding). The uniform topographic gradient of the Saugus–Palmdale grade precludes unequivocal measurement of rod scale errors for the other surveys; however, from a compilation of 1100 km of southern California leveling conducted during 1953–1979, Stein [1981] found a mean rod scale error of  $3 \pm 12$  ppm ( $\pm 1\sigma$ ). This determination yields a standard rod-related error of  $\pm 7$  mm for the 1955, 1961, 1965, and 1971 surveys.

Elevation changes with and without refraction correction are compared in Figure 14. The 60-mm refraction-corrected uplift is independent of the 1964 rod scale error. Strange [1981, Figure 10] previously obtained uplift of 40 mm for the 1955–1965 epoch by using (8) and (9) with more approximate values for  $dT$ . Using (6) and (11), Holdahl [1982, Figure 7] reported uplift of 35 mm for the period 1961–1965. For this same period, we find 50 mm of uplift (Figure 13). The discrepancy may result from Holdahl's selection of the instrument height,  $Z_0$ , of 1.55 m, rather than using  $Z_0 = 1.60$  m as was measured during the experiment. Lowering the instrument height from 1.60 to 1.55 m causes a 10% increase in refraction. Thus the elevation changes presented by Holdahl [1981] appear overcorrected for refraction by about 10% relative to those reported here. Since 1983, the NGS has used  $Z_0 = 1.60$  m for all refraction correction, a value consistent with the results of the field experiment.

Deformation preceding the 1971 San Fernando  $M_L = 6.4$  earthquake is evident in the refraction-corrected elevation changes. The 1955–1961 epoch shows 35 mm of preseismic uplift in the region that sustained 140 mm of coseismic uplift. Deformation preceding the earthquake was previously reported by Thatcher [1976] on the leveling route from Saugus northward to Sandberg and by Strange [1981] along the route from Los Angeles to Saugus (Figure 1). Deformation during the decade after the San Fernando shock (Figure 13) cannot be distinguished from measurement uncertainty.

## CONCLUSIONS

A field test designed to estimate and compare atmospheric refraction in past and current leveling has confirmed the suitability of the balanced-sight equation proposed by Kückkamaki [1938] for the removal of refraction error. The leveling carried out for the experiment meets or exceeds the practices, specifications, and standards of surveys conducted during 1955–1965, which it was intended to reproduce. The +51-mm divergence measured between long- and short-sight leveling (simulating past and current leveling) was reduced to  $-2 \pm 7$  mm after refraction correction. The 23-mm divergence between leveling run forward (toward Palmdale) and backward was reduced to  $+8 \pm 7$  mm after refraction correction.

The nonlinear vertical temperature gradient measured during the experiment indicates that refraction error does not cancel during each instrument setup but, instead, accumulates with a magnitude proportional to the square of the sight length. Because sight lengths were reduced from about 60 to 30 m from 1955 to 1965, the refraction error systematically decreased over time. Historical leveling surveys along the Saugus–Palmdale grade were corrected for both refraction and leveling-rod-scale errors. The corrected leveling exhibits a maximum  $65 \pm 16$  mm of uplift at Palmdale with respect to Saugus during 1955–1965, about 30% of the uplift obtained before refraction models were applied to the leveling. Nontectonic subsidence at Saugus during this period is unlikely to have exceeded  $9 \pm 3$  mm, so the tectonic uplift at Palmdale was  $53 \pm 16$  mm, significant at the 95% confidence level. The corrected data also reveal vertical deformation preceding and accompanying the 1971 San Fernando, California,  $M_L = 6.4$  earthquake.

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