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DISCRIMINATION OF TECTONIC DISPLACEMENT FROM SLOPE-DEPENDENT ERRORS IN GEODETIC LEVELING FROM SOUTHERN CALIFORNIA, 1953-1979

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Abstract. Precise geodetic leveling from southern California carried out between 1953 and 1979 contains linear slope-dependent correlations with a weighted mean value of $(0.3 \pm 2.3) \times 10^{-5}$ times the topographic height difference. This is equivalent to an error of 3 ± 46 mm at the 95% confidence interval over the roughly 1000 m relief of the Transverse Range leveling routes. Linear regression of geodetic tilt onto topographic slope for 1100 km of leveling surveys that are not subject to significant atmospheric refraction error demonstrates that neither the sign nor the magnitude of the correlations changes significantly with time, despite alterations in rod calibration and field procedures during the 1960's. The dominant cause of correlation is errors in the applied rod correction. The errors do not accumulate over several relevels of a route or over distances greater than about 80 km on an individual route; they can be treated as a source of random noise. The rod-corrected uplift from 1953 to 1968 at Bench Mark G54 near Grapevine, a characteristic point on the southern California uplift, north of the San Andreas fault, is 149 ± 18 mm with respect to Saugus (165 ± 9 mm, observed) on a route without significant differential optical refraction. Episodic uplift and collapse along the 100 km ridge route that includes G54 cannot be ascribed to rod or refraction errors, and no more than 48 mm can be caused by ground water withdrawl from the alluvial aquifer beneath Saugus.

Introduction

The search for tools for earthquake prediction has created a need for reliable measurements of displacement and deformation at the earth's surface. Since the rate of deformation appears to be slow and broadly distributed between earthquakes, the detection of aseismic crustal movements demands the highest precision, the longest period of observation, and the greatest areal coverage that can be provided. For measurement of vertical displacements, conventional geodetic leveling best fulfills these requirements. It is a simple, optical, and highly redundant procedure for measuring changes in elevation (Figure 1). It has been carried out in essentially the same manner for almost one hundred years in many places with frequent resurveys. On flat terrain, precise leveling can currently reproduce elevations to within 10 mm over a distance of 100 km [Heiskanen and Moritz, 1967; Bomford, p. 226-280, 1971], while the standard random error for leveling from the first part of this century is only twice as large [Vaníček et al., 1980].

But how accurately can elevation changes be measured using successive leveling surveys in regions of great topographic relief? Recently the accuracy of leveling carried out prior to currently established standards set by the Federal Geodetic Control Committee [1974, 1975] has been challenged by Jackson et al. [1980]. Jackson et al. contend that leveling before 1964 suffers from the accumulation of elevation-dependent errors in excess of one part in ten thousand times the topographic height difference (dH), or greater than 100 mm over 1000 m of relief. Jackson et al. attribute the errors predominantly to mismeasurement of the leveling rods by as much as 1 mm over the 3 m rod length, non-uniform graduation of the rods, and changes in procedures for rod calibration by the National Bureau of Standards.

Castle et al. [1976] assembled 10,000 km of southern California resurveys using the observed elevation differences supplied by the National Geodetic Survey (NGS). These are the measured elevations corrected for the thermal coefficient of expansion for invar and measured linear elongation or contraction of the rods relative to a standard, the rod excess. Castle [1978] concluded that a 70,000 km^2 region within the Transverse Ranges and lying athwart the San Andreas fault's Big Bend underwent 200-400 mm of uplift during 1959-74 (Figure 2a), followed by a partial collapse. If leveling carried out before the uplift commenced were consistently contaminated by elevation-dependent errors as large as a few parts in ten thousand, then the observed change in elevation could be merely an artifact of the

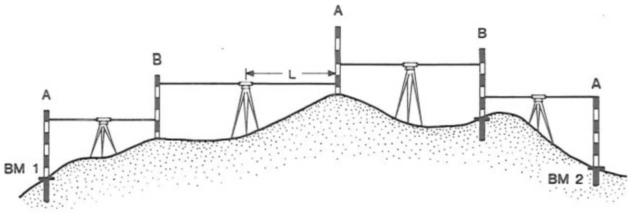


Figure 1. Leveling Procedure: elevation differences between benchmarks (BM's) are measured by sighting adjacent rods with a horizontal telescope, the level instrument. A backsight is made to the A rod, then a foresight is made to the B rod. This is repeated on two scales on each rod. The rods are alternated at every observation; the length of each foresight and backsight pair maintained equal.

error. This would provide one explanation for the similarity between contours of uplift and smoothed topography shown in Figure 2a. If, on the other hand, slope-dependent errors are significantly smaller than 10^{-4} times the height difference, dH, and do not accumulate with time, the observed uplift cannot be ascribed to a linear measurement error. If real, uplift and subsidence of the Transverse Ranges would then demonstrate that these youthful mountains are actively growing and deforming, both during and between earthquakes.

Using a number of independent tests, the contention of Jackson et al. [1980] will be probed on 1100 km of levels carried out between 1953 and 1979 within the uplifted region. Because Strange [1981] presents evidence that accumulating differential refraction, a non-linear elevation-dependent error, can significantly modify observed elevations, this study will be confined to leveling routes where differential refraction must be minimal.

Strategy

The intent of this statistical analysis is to determine the maximum magnitude of accumulating leveling errors, and to remove these errors from a representative and critical resurveyed leveling route. It is assumed in the analysis that elevations measured from any given leveling survey, n, contain: (1) a linear, rod-related error, (2) a non-linear atmospheric refraction error, and (3) real earth movement, all of the same order of magnitude. The rod error can be approximated by

$$dH_n = (1+e_n)dH, \qquad n = 1,2 \tag{1}$$

where e is the rod excess; e = measured minus true rod length. Differencing the two surveys to obtain elevation change, dh,

$$dh = dH_{n+1} - dH_n = (e_{n+1} - e_n)dH \tag{2}$$

$$\text{or } dh = e_{net}\, dH \tag{3}$$

where $e_{net} = e_{n+1} - e_n$.

The simplest expression that is commonly invoked for the atmospheric refraction error, from Kukkamäki [1938] for resurveyed elevations is

$$dh = \gamma^*[(L_{n+1}{}^2 \Delta T_{n+1} - L_n{}^2 \Delta T_n)]dH, \tag{4}$$

where L is the sight length between rod and instrument (Figure 1), ΔT is a linear approximation of the vertical temperature gradient along the line of sight, and γ^* is a physical constant. Since observations of the thermal gradient and hence ΔT do not exist for historical leveling, the assumption is made here that $\Delta T_1 \approx \Delta T_2$, as the squared sight length term will in any event dominate. Then (4) becomes

$$dh = \gamma(L_{n+1}{}^2 - L_n{}^2)dH \tag{5}$$

where $\gamma = \gamma^* \Delta T$.

Because any profile of elevation, or elevation change from resurvey, may exhibit a trend - a mean slope not equal to zero - bench marks spaced far apart along the leveling route will be more likely to show both a larger dh and dH than those closely spaced. To remove this bias, tilt measured by resurvey (dh/dx) can be compared with topographic slope or grade (dH/dx), where dx is the distance

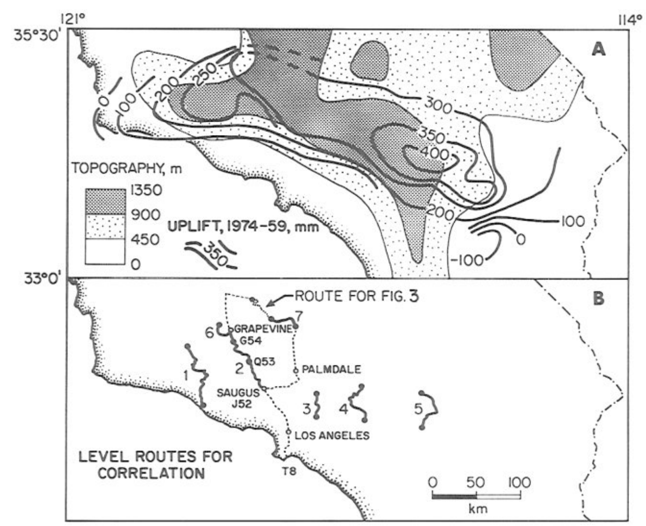


Figure 2. A. Smoothed contours of uplift from 1959 to 1974 from Castle [1978] superposed on smoothed Southern California topography shaded. Note the striking similarity between uplift and topography, except in the south (the Salton Sea and Peninsular Range), and in the north (Sierra Nevada Range). B. Level routes. The dashed circuit through Saugus, Bakersfield, and Palmdale is used for calculation of misclosures. Route 2 is called Ridge in the text.

between adjacent BM's; tilt and slope are of course independent of BM-spacing. Dividing eqns. (3) and (5) by dx and adding to these expressions uncorrelated residual tilt, ε, results in a combined simplified expression,

$$\frac{dh}{dx} = [e_{net} + \gamma(L_{n+1}^{2}-L_{n}^{2})]\frac{dH}{dx} + \varepsilon, \; n=1,2 \quad (6)$$

Eqn. (6) must be solved under conditions mutually satisfactory for e_{net}, γ, and ε. Resolution of the rod term, e_{net}, demands that (6) be evaluated only for resurveys where rods have not been changed. If in survey n or n+1, more than one pair of rods is used, the correlation associated with each rod pair difference can be obscured in the regression. Also, since e_{net} is linear, successful correlation requires a large variance in dH/dx (slope) to overcome sources of random error, soil-, and BM-instability. This requires rough topography, with large excursions from the mean slope. Discrimination of the refraction term, on the other hand, requires evaluation over constant atmospheric and ground conditions to ensure that γ is constant; this is probably optimized over a uniform slope. Also, the variance of $(L^{2}-L^{2})$ must be large, which requires gentle slopes ($\leq$ 0.02 or 2%). For grades less than 2%, L will not be constrained by the useable rod height (2.5 m) to be the same for both surveys n and n+1.

Thus, despite gross simplifications made to obtain eqn. (6), the conditions necessary for multiple linear regression to find e_{net} and γ are incompatible, because to solve for e_{net} requires a large variance in dH/dx, the slope, while to find γ requires uniform and gentle dH/dx. By pursuing limiting cases of (6), two strategies are possible. One that will be employed in this paper is to consider only routes where both $\Delta T_{n+1} \approx \Delta T_{n}$ and $L_{n+1} \approx L_{n}$, so that γ, and hence refraction error, becomes negligible; this requires rough, steep topography. The alternative is to take cases where dH/dx is constant, in which case e_{net} and ε become indistinguishable. This approach suffers because real earth movement cannot be segregated from rod error. Also, the assumption of nearly constant ΔT becomes critical to the resolution of γ. If $L_{n} = L_{n+1}$, (6) simplifies to

$$\frac{dh}{dx} = e_{net}\frac{dH}{dx} + \varepsilon, \; n=1,2 \qquad (7)$$

A linear least-squares regression of tilt onto topographic slope is performed for each resurveyed segment. No segment contains any rod changes that mask the correlation (e.g., containing both positively and negatively correlated segments). Because random leveling errors grow with the square root of distance, tilt or elevation change can be more accurately measured from benchmarks farther apart [Bomford, 1971]. To give more weight to the better data, weights, w, are assigned, where w = dx, the distance between benchmarks. Both the weighted and unweighted coefficients of correlation, r, are calculated. To favor the identification of a correlation, the highest of the weighted and unweighted absolute value of r is chosen. The significance of r is checked by an equal-tails test of the null (or no correlation) hypothesis. Refer to the Appendix and Stein [1980] for a more detailed discussion of the weighted linear regression, and the basis for assignment of weights. Because the least-squares regression is not robust, outliers, or sections with extreme values of tilt or slope, must be removed. If this is not done, a significant line can be fit by connecting a cluster of points with one extreme point. However, no significant line will be found if this one point is removed. From 10 to 20% of the total benchmark population is removed; this includes marks that differ by 5 mm from adjacent marks, outliers, and in some cases additional marks whose removal will strengthen the correlation. If r is significant at the 95% level of confidence, the correlation is accepted as an error, e_{net}. If the values of e_{net} for adjacent segments do not differ by more than one standard deviation, the releveled segments are correlated together in the same regression, and retained as one segment if the significance of the correlation increases. In this manner the rod pair responsible for the correlation can be isolated from rod changes that have no effect.

To estimate the topographic roughness required for significant correlation, a 30 km segment from a railroad grade in the Transverse Ranges has been isolated because its slope is almost uniform (Figure 3). If the dh profile were the product of elevation-dependent error, it should display a constant tilt. However, the error cannot be found simply by dividing the mean tilt by the (constant) slope because that is the long wavelength (30-km) signal; it is the mark-to-mark correlation that is sought. To reduce the noise in this as in any elevation change profile, some benchmarks must be eliminated. If the mark rejection criterion of Jackson et al. [1980] is stiffened to favor correlation, such that any mark that differs 5 mm from both its neighbors is eliminated (they use a 10 mm difference), then the standard deviation about the mean tilt of the remaining 80% of the benchmarks shrinks from 2 to 1 μrad (10^{-6} radian). To find an elevation-dependent error, e_{net}, of 10^{-4}, a 2% grade (0.02 slope) range is required to overcome the bench mark scatter; a 10% range in grade of the topography is needed to resolve a 5×10^{-5} error. Therefore, only leveling routes with grades that vary by about 10% will be selected for regression, and only segments that contain this range can be isolated from the remainder of the survey route. This criterion effectively eliminates all railroad grades from consideration because their slopes usually vary by no more than 2%, and hence, lack the topographic variation necessary for regression. So while railroad routes can contain useful leveling data, they cannot be used to

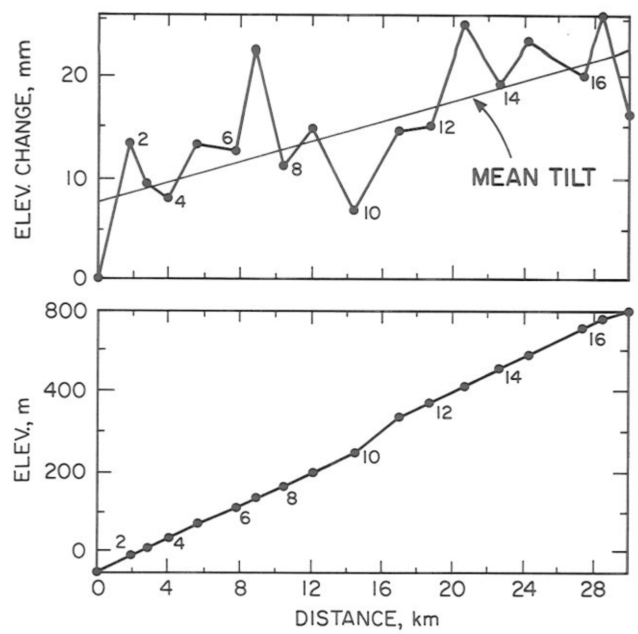


Figure 3. Variation of tilt over constant slope - this 30 km railroad segment is shown in Figure 2b. Since the topographic slope is almost uniform (lower profile), there is no signal to correlate with tilt (upper profile). The roughness or variation in slope in the lower profile must exceed that of the upper profile for significant correlation.

detect significant elevation-dependent systematic error.

Results

1. The Ridge Route

In a number of respects the 100 km leveling route from Saugus to Grapevine is ideally suited to test for real tilt and rod errors in eqn. (7); it will be referred to as Ridge in this report (Figure 2b). Its full length has been resurveyed six times from 1953 to 1974, spanning the years during which observed uplift and a change in rod calibration and leveling procedures took place that Jackson et al. [1980] contend is significant. Elevation changes can be tied to the long-term tide gauge station at San Pedro through benchmark Tidal 8 (T8 in Figure 2b) as Castle et al. [1976] have done. The route transects the southern California uplift identified by Castle et al. [1976]. Ridge develops a 1000 m gain and loss in elevation, with the majority of section slopes distributed between 7 and -7%. The sight lengths for Ridge have been short and roughly constant for all surveys, (the mean sight length, L, varies from 20-25 m; see Figure 1), and each survey was performed in the Spring. Both of these factors reduce the potential error caused by differential optical refraction to below 10 mm [Strange, 1981]. Most of the benchmarks are emplaced in 10 m.y. old consolidated sediments and 100-200 m.y. old weathered granites [Jenkins, 1975]. North of Grapevine lies a well-documented region of major pumping-induced subsidence that cannot be subjected to analysis [Lofgren, 1975].

Profiles of topography, observed elevation change, and adjusted elevation change are displayed for successive survey intervals in Figures 4a through 8a. Saugus is at 0 km, and Grapevine is located at 95 km. Except for the first (1953) and last (1974) surveys, each survey is used twice, as the latest and earliest survey of the paired differences (e.g., 1965-64, 1968-65). The plot of tilt as a function of slope for each survey difference follows as Figures 4b through 8b. Rod changes are demarked at vertical lines, and since the rods used in either the early or late survey may change, the year of the change is indicated beneath the upper profile.

During the first and longest resurvey interval, 1964-53, tilt is not correlated with slope in rod segment A (Figure 4a). 80 mm of uplift takes place over essentially flat terrain, whereas the elevation increases only 10 mm over a region of 600 m relief. Since segment A is uncorrelated, it is left unadjusted in the top profile of Figure 4a. Both segments B and C are positively correlated, as can be seen by inspection, and yield a very large combined slope-dependent error, e_{net}, of $(13.2 \pm 1.1) \times 10^{-5}$ (Figure 4b). The correlation coefficient for the weighted regression of tilt onto slope, r, is 0.84, equivalent to 99% confidence that the tilt and slope are correlated. In other words, 84% of the observed tilts measured between benchmarks are equal to $(13.2 \pm 1.1) \times 10^{-5}$ times the topographic slope between those marks.

To remove the correlated error, e_{net}, from the observed elevation change profile, the correlated component of tilt, which is equal to e_{net} times the slope, is subtracted from the observed tilt. This operation is performed for each section in the segment, and the resulting profile is plotted as the adjusted elevation change. This adjustment need not remove the entire observed tilt, since some portion of the tilt may be uncorrelated with topography. Rather, the adjusted elevation change has no dependence on slope; a regression of adjusted tilt onto slope would produce both e_{net} and r equal to zero. For this 11 year interval (1964-53), the observed and adjusted elevation changes display nearly the same net uplift with respect to Saugus, despite the large error in segments B and C. This is because most of the elevation changes take place in the absence of topographic relief, while the correlated segment rises only an additional 300 m to the peak elevation. In contrast to the southern end of the route, the elevation of Grapevine (at the 95-km position, Figure 4a) is 90 mm higher in the adjusted profile relative to the observed profile.

During the survey interval, 1965-64, the same segments that displayed a large positive correlation in 1964-53 show an almost equally large negative correlation (36-98 km distance, Figure 5). Once again, segment A (0-36 km) remains uncorrelated, with nearly uniform tilt over a great range of slopes. The most straightforward explanation for the reversal in sign of e_{net} for segments at 36-98 km is that the 1964 tilts are positively correlated, and the 1953 and 1965 tilts are relatively free of correlations.

Rod segments A and B have nearly the same correlation in the 1968-65 survey interval (Figure 6). If segments A and B contain elevation-dependent errors, the value of the error does not differ between them by more than 1.5×10^{-5}, which is roughly the limit of resolution. The high amplitude displacements at 25-35 km distance (Figure 6a, elevation change profile) cannot be correlated with elevation; the excursion in tilt over these sections is much larger than for the remainder of the segments. However, if those five benchmarks are removed from the regression as part of the 20% mark deletion, a correlation with 99% confidence can be obtained. In addition to a real elevation-dependent correlation that persists for 75 km, there is a confined anomaly. After correction of the entire segment including the anomaly, the form of the anomalous region is essentially preserved in the adjusted elevation change profile. The adjusted uplift is about 20 mm less than the observed uplift during the 1968-65 resurvey interval.

The 1971 survey was run soon after the M6.4 San Fernando earthquake. Because the first 20 km of Ridge lies within the aftershock zone, this portion of the route for the interval 1971-68 underwent seismic displacement that overwhelms slope-dependent correlation [Castle et al., 1975]. So while the southernmost portion of segment A (0-7 km, Figure 7a) cannot be correlated, the remaining two portions of segment A (16-41 km and 46-77 km) yield a correlation of $(-6.8 \pm 0.8) \times 10^{-5}$ (Figure 7b). This value of e_{net} has been removed from all portions of segment A in the adjusted profile. Segments A and B alternate in the profiles (Figure 7a) because two rod sets were periodically exchanged during the 1971 survey. The adjusted elevation change during this interval shows 35 mm greater uplift than the apparent elevation change.

In the final resurvey interval, 1974-71, rod segment A is again correlated, but this time positively (Figure 8). The negative 1971-68 and positive 1974-71 correlations indicate that the 1971 survey within segment A is negatively correlated. Its value for e_{net}, for comparison, is about one-third of the 1964 error.

Net uplift from 1953 to 1974. The observed and adjusted elevation changes for the entire 21-year interval can now be compared by summing the changes at a few benchmarks common to all resurveys (Figure 9). The total observed elevation change near Grapevine, benchmark G54 at the 82-km position, with respect to Saugus, J52 at 0 km, is 121 ± 9 mm. Here the standard deviations represent the random error. The total adjusted elevation change for the same interval is 128 ± 24 mm at Grapevine with respect to Saugus, where the larger standard deviation contains both the random error and the uncertainty of adjustment. The adjustment error is calculated from the standard deviation of e_{net} for successive resurvey intervals. The observed and adjusted values of uplift agree closely despite the fact that 65% of the 470 km of releveled segments in Ridge produce elevation-dependent correlations with 99% confidence. How can this happen? The good agreement arises because the mean value of e_{net} averaged over successive intervals is nearly zero, and because many of the larger tilts are uncorrelated.

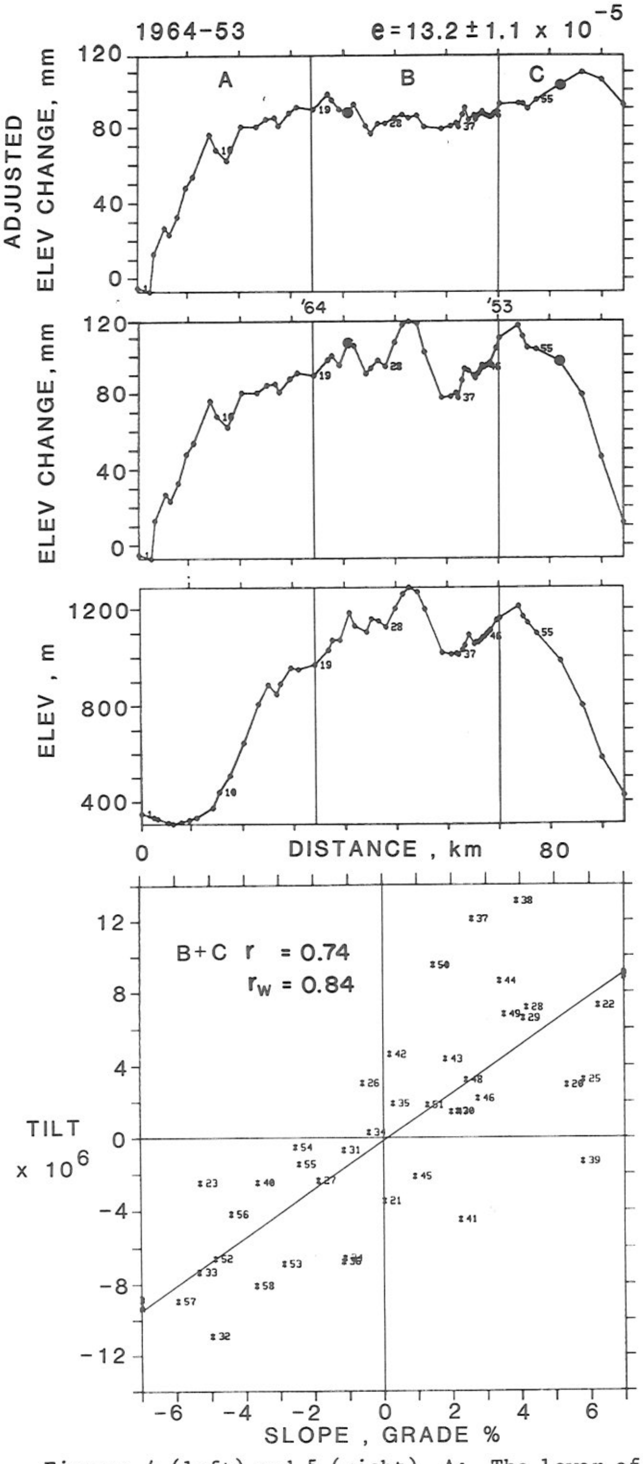
2. Compilation of correlations for all routes

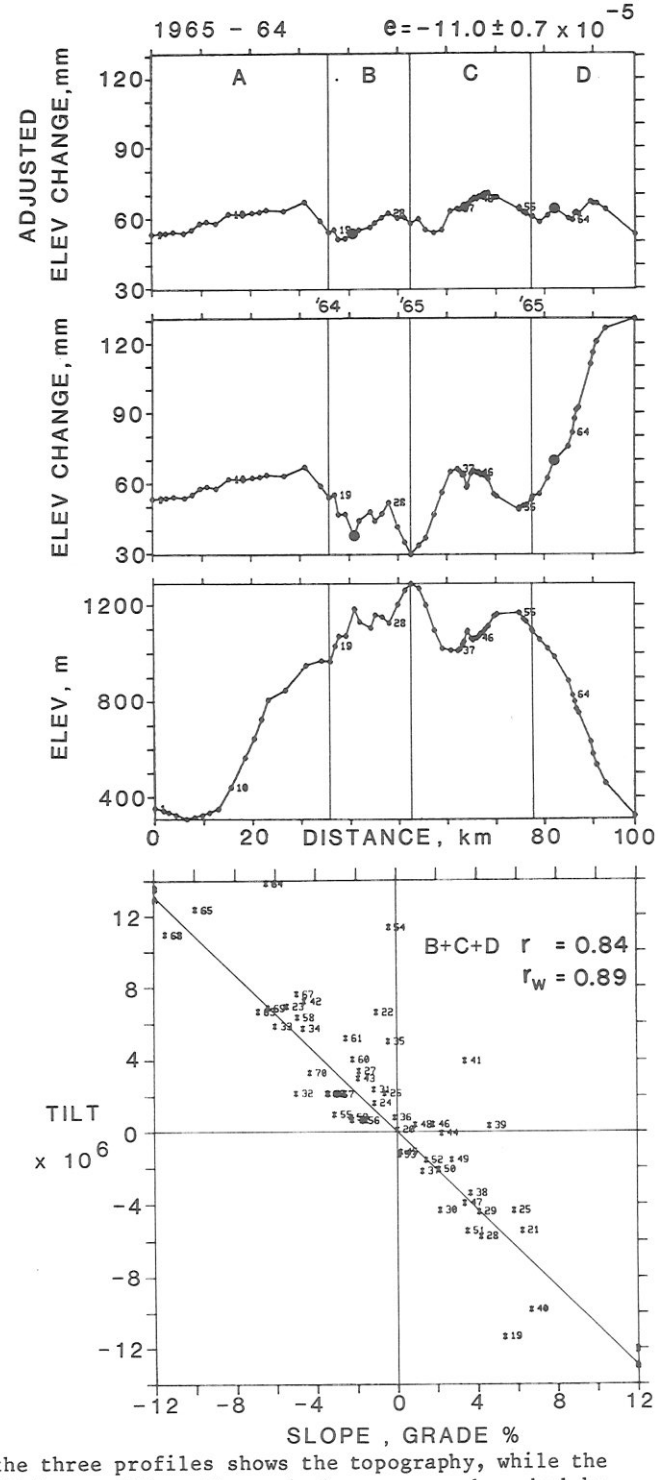
1700 km of leveling surveys were investigated for correlation. 1100 km of the levels proved to contain both the topographic variation and the absence of regions of pumping-induced subsidence, necessary for regression. The route locations are shown in Figure 2b and a summary of these correlations is presented in Figure 10. The resurvey intervals for rod segments are grouped roughly into chronological order. Since each correlation derives from two differenced surveys and the resurvey interval varies, the temporal sequence is not exact. Note that e_{net} of eqn. (7) is the combined error from two rod pairs, one from each survey. Both significant (r or $r_w \geq 95\%$) and insignificant correlations are plotted and used in computations. The variance of correlation coefficients by maximum likelihood is used to calculate the weighted mean, μ, and variance, σ_o^2, of the population. This yields $\mu = 0.34 \times 10^{-5}$, and $\sigma_o^2 = 5.2 \times 10^{-5}$, which means that the mean true correlation is $0.3 \pm 4.6 \times 10^{-5} \times dH$, at the 95% confidence interval.

Discussion

The near-zero mean error and the general lack of a significant change in sign or magnitude of the error with time form a crucial finding of this work. The errors are equally abundant and nearly equal in magnitude before, during, and following the period of observed uplift in southern California, 1959 to 1968. Had rods tended to shorten with time, and this length change gone undetected by calibration, the mean correlation would have been positive, rather than close to zero. This is because every value of e_{net} derives from a difference of a later rod pair from an earlier pair. The same circumstances would result if the rods maintained stability but the length measured from calibration erroneously increased with time. If during a particular epoch, rods shortened or calibrations increased, the mean value of e_{net} for that period would be positive. Neither effect emerges in Figure 10.

The mean error and population standard devi-





Figures 4 (left) and 5 (right). A: The lower of the three profiles shows the topography, while the elevation change from resurvey is shown in the middle profile. The rod changes are demarked by vertical lines, with the year of each change indicated. If a segment is uncorrelated, it is reproduced in the adjusted top profile, whereas if it is correlated at r or $r_w \geq$ 95% interval of confidence, the correlation is removed in the upper profile. The slope of the regression, e_{net}, is shown in the upper right. Large dots are BM's shown in Figure 9. B: Plot of tilt as a function of slope for the correlated segments.

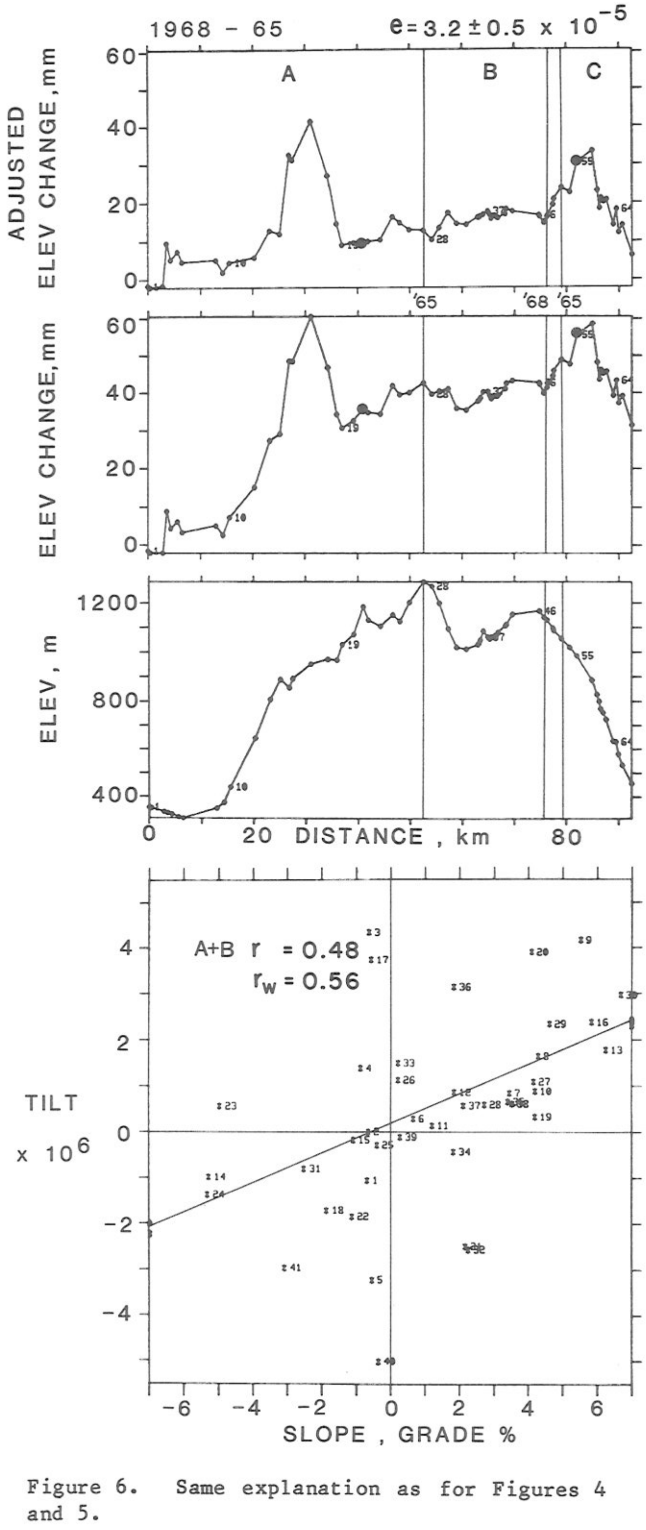


Figure 6. Same explanation as for Figures 4 and 5.

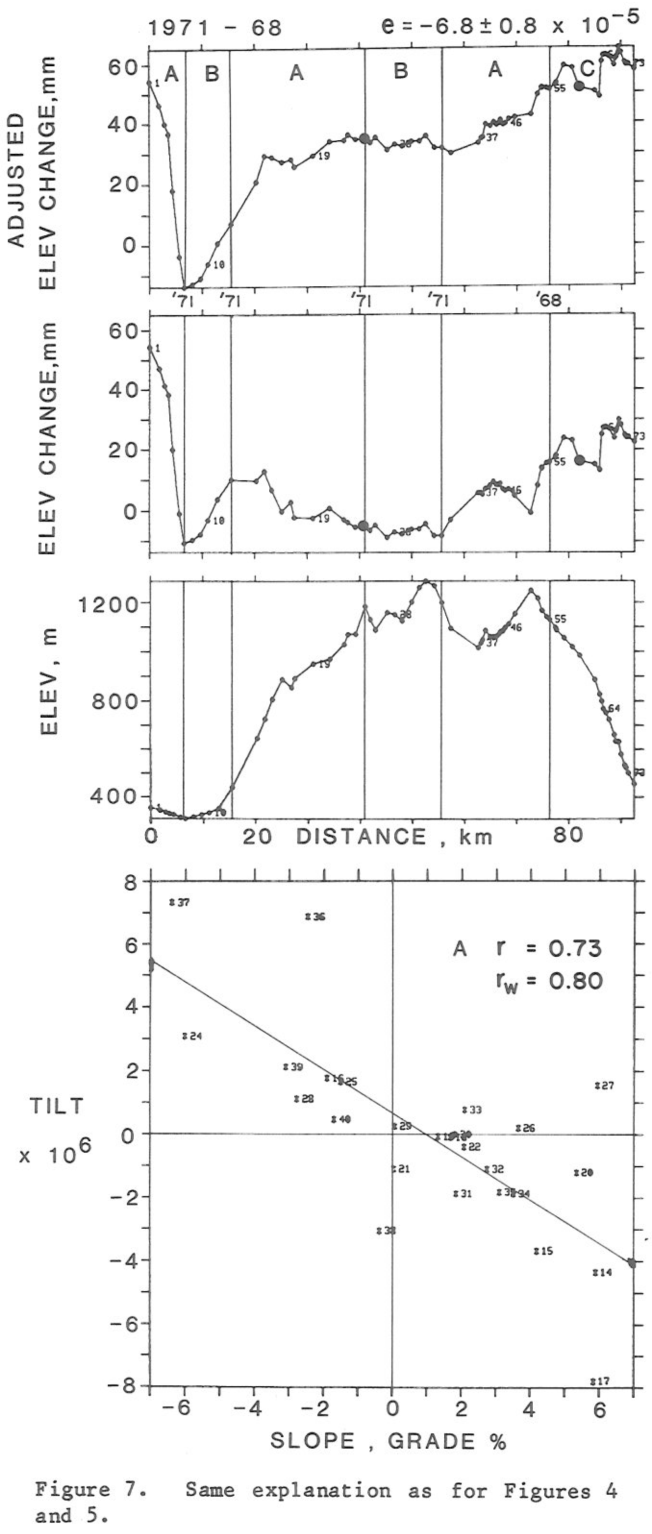


Figure 7. Same explanation as for Figures 4 and 5.

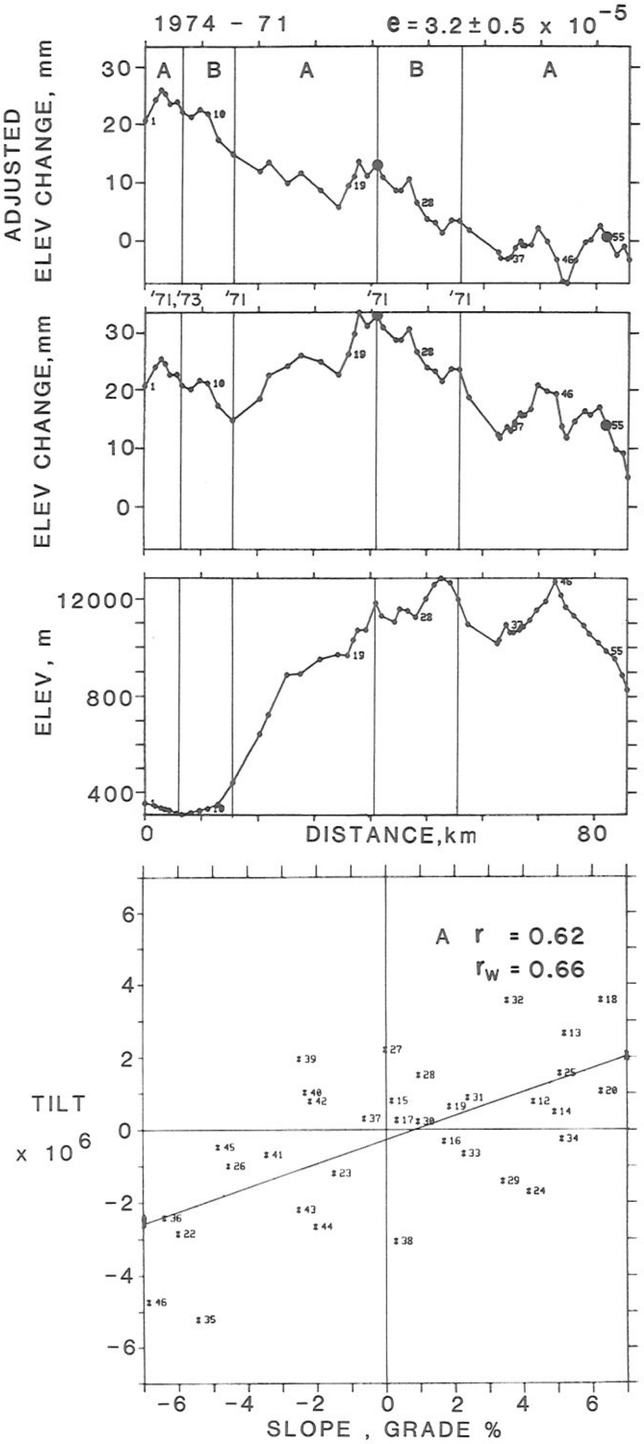


Figure 8. Same explanation as for Figures 4 and 5.

ation is $(0.3 \pm 2.3) \times 10^{-5} \times dH$. Because the variance of $\pm 5.2 \times 10^{-5}$ produces no more than 50 mm of artificial uplift over the maximum 1000 m relief, over three of these errors must accumulate with time to equal the 149 mm of uplift at G54 in 1968. In other words, there is greater than 99% confidence that the uplift shown in Figure 9 is unrelated to the systematic error tested for in eqn. (7). Since positive and negative errors are almost equally distributed and persist for distances less than 80 km, these errors do not accumulate. The observed elevation changes in steep terrain should therefore be accurate to 50 mm (2σ) in most cases, and to 80 mm (3σ) in almost all cases.

The two unusually large errors visible in Figure 10 have the same rods in common (rods 312-268, -274), used in some of the 1964 leveling. Jackson et al. [1980] cite rod 268 as typical, displaying non-uniform elongation that is not removed by the linear rod correction used to reduce the measured elevation changes. When compared to the one hundred other National Geodetic Survey rods calibrated by the National Bureau of Standards after 1964, it is clear that rod 268 is exceptional: the standard error of a linear fit to the rod differs by 3σ from the population mean [see Mark et al., 1981, Figure 5]. For both the Ridge and the Saugus to Palmdale routes there are surveys before and after 1964. The large positive (1964-53) and negative (1965-64) correlations almost cancel, leaving almost no cumulative effect on the observed elevation change. Specifically, the Ridge error for 1964-53 is $(13.2 \pm 1.1) \times 10^{-5}$, while that for the succeeding interval, 1965-64, is $(-10.8 \pm 0.7) \times 10^{-5}$. Adding the two intervals gives the net correlated error for 1965-53, $(2.4 \pm 1.3) \times 10^{-5}$, within the mean standard error for all resurveys.

Significance of residual tilt

The mean residual tilt, ε, of eqn. (7) is rarely found to be statistically significant for a regression regardless of its correlation coefficient, r. This can be seen by inspection of Figures 4 though 8; the y-intercept does not differ significantly from zero. Physically, this means that over a leveling segment that can be subjected to regression, the tilt is most often not uniform. Only rarely does tilt between any three successive bench marks exceed 2×10^{-6} or 2 μradians. The standard error for ε of the regressions is in all cases greater than $\pm 1 \times 10^{-6}$, or ± 1 μradian. For the 30 km average segment length in Figure 10, a 60 mm uplift of one end with respect to the other would be required to achieve 95% confidence as a real tilt. For Ridge, it can be seen from Figure 9 that only during the first time interval does such a large elevation change take place. Thus while the residual displacements shown in Figure 9 are essentially free of both rod and refraction errors, and while the resultant uplift is significantly larger than both expected random errors and the error of adjustment, the mean tilt is not significant.

Tests for rod-related errors

1. If the correlations are related to leveling rods, the error, e_{net}, should change significantly when rods are changed. For the Ridge resurveys, this proved to be the case for each of the five resurvey intervals, although the error did not differ significantly for every rod change. Five other resurvey intervals display significant changes in e_{net} with rod change, along routes 1, 2, 4, 5, and 7 (Figure 2b). These relevels span the years 1953 to 1979. Note that even if the entire rod population contained significant errors, the correlations would not always change detectably with change of rods. This is because the errors of some adjacent rods may differ by less than the correlation resolution from a perfect rod standard.

2. The error contribution of a specific rod set, e_n, can be distinguished from e_{net} under special circumstances. For route 5 (Figure 2b), correlated errors with 99% confidence are obtained for the resurvey intervals 1979-78 and 1978-76 over 23-32 sections with a grade range of 8%. The rods used for these surveys overlap two to five benchmarks leveled with different rods. Four groups of resurveyed sections exist, called lap sections, where a number of benchmarks were releveled with the changed rods rather than the more common procedure where each rod pair shares only one common benchmark. The elevation-dependent error for each lap section can be

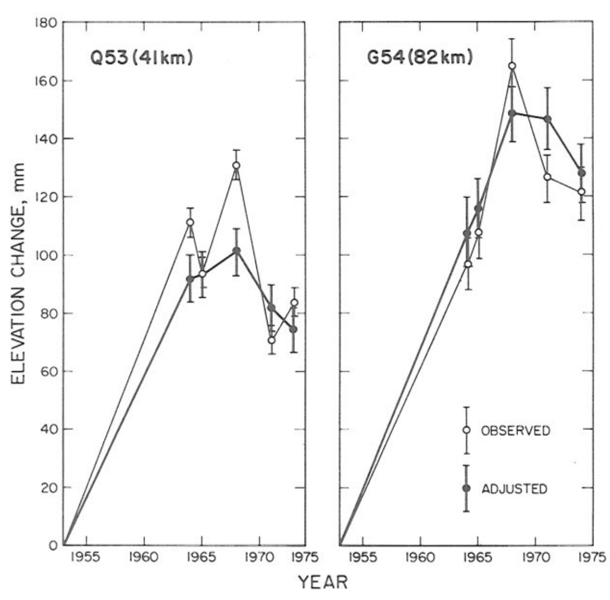


Figure 9. Uplift histories for two representative BM's along the Ridge route. The BM's are indicated by large dots in Figures 4-8. The observed and adjusted values are usually within one standard deviation.

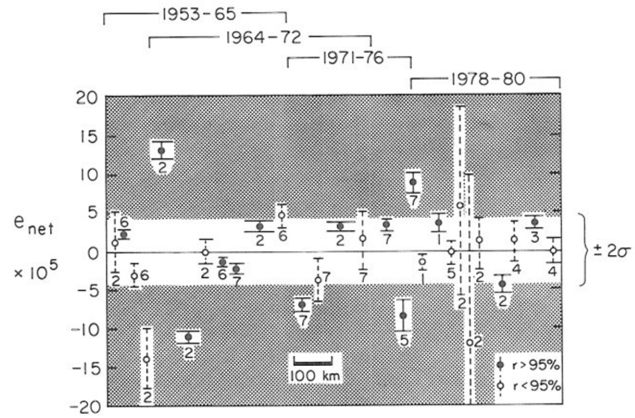


Figure 10. Summary plot of the slope-dependent correlations from 1100 km of relevels, arranged chronologically. Routes without significant correlations are dashed. The width of the brackets indicates the distance over which the correlation is maintained. Numbers correspond to the routes in Figure 2b.

reduced to five approximate linear equations for the errors of five rod pairs (standard deviations of about $\pm$ 2 x 10^{-5}):

$$
\begin{aligned}
A-B &= -10.0 \times 10^{-5} \\
A-D &= -\ 8.8 \times 10^{-5} \\
A-E &= -\ 8.3 \times 10^{-5} \\
D-B &= \ \ 0.6 \times 10^{-5} \\
D-C &= \ \ 0.4 \times 10^{-5}
\end{aligned}
$$

where each letter corresponds to a separate rod pair. Solving the simultaneous equations yields

$$A = -9.0 \pm 2.0 \times 10^{-5}$$

$$B = C = D = E = 0 \pm 2.0 \times 10^{-5}$$

A large negative correlation is found each time rod pair A is used with D and B, whereas B and D exhibit negligible correlations when compared to each other or two other rods. The rod pair A (316-132180, -87849) error, together with the 1964 and 1971 rod pair errors from Ridge that could also be isolated from e_{net}, demonstrate that these errors are neither time- nor location-dependent, but can be assigned to specific rods.

3. A third test for rod-related errors can be designed by considering the complete 354 km level circuit that contains Ridge, Saugus and Palmdale (Figure 2b, dashed). This circuit has been leveled within two years on four occasions (1953/55, 1964/65, 1972/74, and 1978), both before and after the period of observed uplift. Rods were changed from 9 to 24 times during each circuit. If no period contains elevation-dependent errors significantly larger than the mean error, the misclosures, or differences between initial and final elevations of the Saugus terminus of the

route, should be similar in magnitude. Further, if the errors are fixed to the rods, the errors should randomize and the misclosure should be of the same order as the mean rod error. The mean misclosure is -10 ± 36 mm, consistent with the observed elevation-dependent error of 3 ± 45 mm over the 1000 m elevation difference (Table 1). This test holds regardless of the impact of differential refraction, since surveys with dissimilar sight lengths are not differenced, and because temperatures do not vary significantly during the survey. (Strange [1981] applied about a 10-30 mm refraction correction to the misclosures.)

4. Correlations that change significantly without an associated rod change constitute a strong test for errors that are independent of rods. Because a segment must be about 15 km long to achieve a correlation with 95% confidence, this test can only be performed in special cases where rod changes were infrequent. No cases have been found where a 15 km segment adjacent to a correlated rod segment displays the same value of e_{net} within one standard deviation at 95% confidence.

5. Another test for errors independent of rods can be performed by isolating segments where the same rod pairs were used in both surveys of a route, eliminating the effect of rods regardless of the rod error, under the assumption that no rod strain and no real earth movement took place between surveys. Only two such cases have been located in the 1700 km searched, both 6-10 km segments leveled in the early 1970's (Figure 11). In neither case does the elevation change for the resurveys differ by more than 2 mm over the length of the segment. The assumptions therefore appear justified unless rod strain balanced rod movement, suggesting that no errors emerge in the absence of rod differences.

Sources of rod-related errors

A number of factors contribute to discrepancies in the measurement of and correction for rod excess, or the difference between actual and measured rod lengths.

Improperly encoded calibrations. The National Geodetic Survey produces a computer encoded list

TABLE 1. Circuit Misclosures

Years	Rod Changes	Orthometrically corrected misclosure, mm
1953/55	9	+24
1964/65	13	-70
1972/74	9	+11
1978	24	-5
Mean	14	-10 ± 36

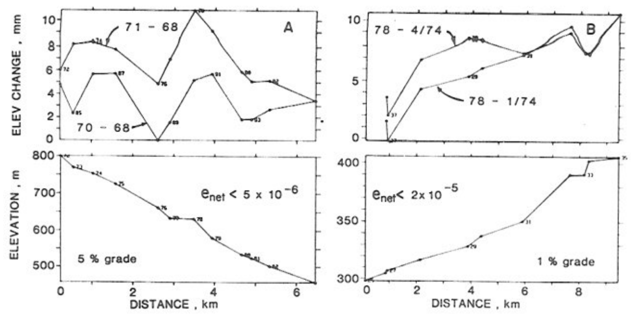


Figure 11. Segments surveyed twice with the same rod pair show no slope-dependent correlations $> 2 \times 10^{-5}$.

of rod calibration records, the RIF (Rod and Instrument File) that is used to correct field elevations for measured rod excess. There are internal inconsistencies in this list that result in improper corrections. For example, the encoded rod excess for Los Angeles County rods 315-95, -96, calibrated in 1977, is -3.1×10^{-5}, inconsistent with the listed calibration values that indicate an excess of -0.3×10^{-5}. The error deduced from statistical analysis for leveling with the RIF calibration is $(-5.1 \pm 0.9) \times 10^{-5}$ (Figure 12), only slightly larger than the encoding error.

Improperly calibrated rods. The points of observation for calibration have never been standardized for first order leveling, and at least three agencies have performed the calibrations: the National Bureau of Standards, Navy Gauge and Standards Center, U. S. Geological Survey [Kumar and Poetzsche, 1980]. Some rods are observed for calibration at 200 mm from the footplate, despite the fact that the first order leveling procedure precludes sighting the rod below 500 mm [Federal Geodetic Control Committee, 1974, 1975]. This leads to a probable 3.4×10^{-5} error in calculated rod excess for NGS rod 316-87849, calibrated in 1977 (Figure 13).

Damage to rod in use. Undoubtedly some rods are damaged during leveling so that the pre-survey calibration is no longer appropriate for correction of field elevations. Rod 312-268, used in the 1964 survey of Ridge, shows both a large and non-uniform excess of $(8 \pm 7) \times 10^{-5}$ in its post-survey 1965 calibration. However, the statistically measured excess for this rod and its mate is $(-12.4 \pm 0.7) \times 10^{-5}$ (Figures 5 and 6). Since both the calibration and statistically calculated errors are large but different, damage appears likely. Unfortunately, the rod serial number is attached only to the rod frame; the invar tape is periodically changed and its strain or damage cannot be traced.

Thermal coefficient of expansion for invar. The values of TCE for invar range from $(-2.5$ to $+3.0) \times 10^{-6}/°C$, and standard field temperatures range from about 10-30°C [Kumar and Poetzschke,

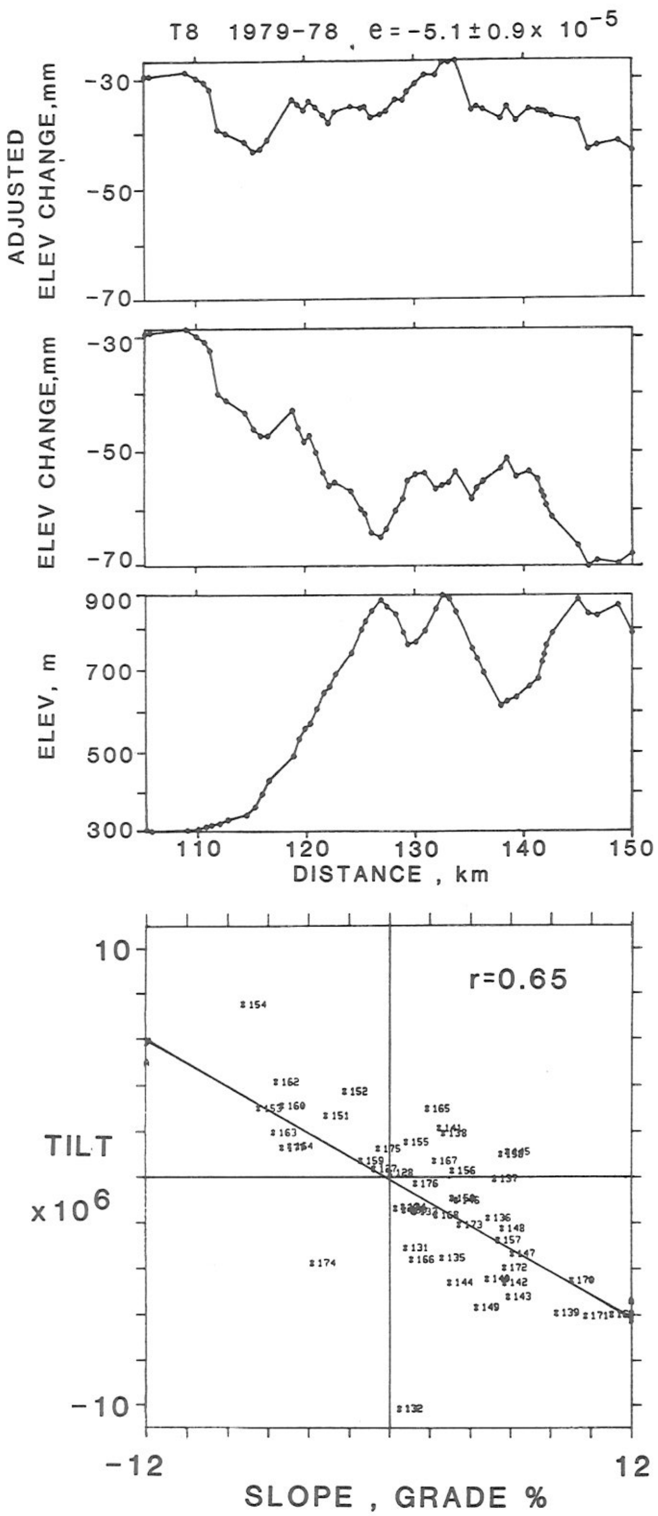


Figure 12. 50 km segment of 200 km route from T8 to G54 (route shown in Figure 2b). The encoding error in the RIF is 3.1×10^{-5}.

1980]. The NGS assigns a value of $0.8 \times 10^{-6}/°C$ for the same commercial rod (Kern) that the USGS applies $2.3 \times 10^{-6}/°C$, for temperature corrections. This can lead to discrepancies of 2.5×10^{-5} under the temperature range expected during leveling in southern California.

Thus at least some of the errors shown in Figure 10 are related to encoding, computation, and assumptions about thermal response, rather than miscalibration. What is important in considering these factors is that they are not systematic in time: none cause most older rods to be longer than their nominal length, or cause current rods to be shorter than their nominal length. Neither the statistical evidence compiled for Ridge (Figures 4-8), the 1100 km of relevels from 1953-1979 (summarized in Figure 10), nor an examination of potential causes of the elevation-dependent correlations, indicates that the errors are systematic in time, or confined to a particular location. The errors can therefore be treated as a source of random noise.

Comparison with other studies

Leveling rod errors. Strange [1980] produced a maximum estimate of leveling rod errors by comparing 64 resurveys over 17 leveling routes with varying topography. The elevation, dH_n, of the endpoint of each resurveyed route was subtracted from the initial endpoint elevation, dH_1. This method measures real earth movement as part of the error. Nevertheless, the 95% confidence error reported was about $6 \times 10^{-5} \times dH$, which is only slightly larger than that obtained in this analysis, where rod errors and earth movement are separated.

The central contention of Jackson and Lee [1979] and Jackson et al. [1980] is that the temporal change in elevation measured from resurveys across the southern California uplift correlates with elevation at the kilometer scale; that is, the apparent elevation change (dh) between surveys mimics the elevation difference (dH) from one benchmark to the next. Jackson et al. [1980, their Figure 1] demonstrate this correlation with a plot of the spatial variation in incremental elevation change, d(dh)/dx, with the incremental change in topography, d(dH)/dx. The technique of Jackson et al. has been applied to the survey years 1965-64 over the segment Saugus to Palmdale where 160 mm observed elevation change is concentrated (route location, Figure 2b; plot, Figure 14a). In Figure 14b, the same technique is employed to correlate a straight line - a uniform tilt with endpoints as in the observed elevation change, and with benchmarks spaced along the route as in reality - with the true topography. Because the tilt of the straight line is uniform, there can be no benchmark-to-benchmark correlation with topography in Figure 14b. Despite this, both Figures 14a and 14b display what appear to be impressive correlations of similar magnitude. Why

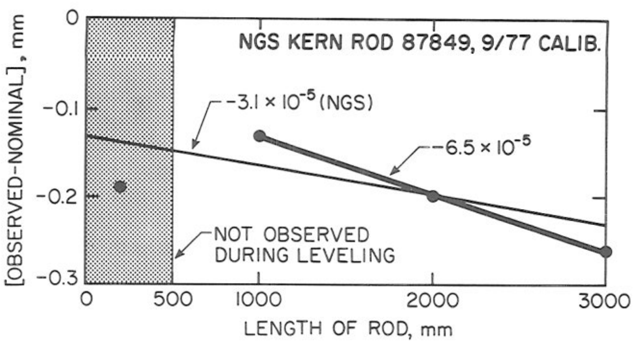


Figure 13. Calibration over the useable length of rod differs by 3.4×10^{-5} from the NGS RIF applied rod excess.

does the technique of Jackson et al. [1980] fail to discriminate between topography correlated to actual leveling surveys, and topography correlated to a uniform tilt devoid of any short wavelength (mark-to-mark) signal? The problem lies in the fact that the topography, the real elevation change, and the uniform tilt have precisely the same variation in benchmark spacing, as Stein [1980] and Mark et al. [1981] have pointed out, a property common to all leveling data. Because the elevation, elevation change, and straight line all have a trend (a mean slope not equal to 0), benchmarks spaced farther apart will tend to show both larger dh and dH than those more closely spaced, though their tilt (dh/dx) and slope (dH/dx) are of course independent of spacing. This can be seen in Figure 14c, which shows the two profiles that are correlated in Figure 14b. The benchmark spacing for a leveling route will always vary more than an order of magnitude, in this case from 0.02-4.27 km, so that unless the values of dh and dH are normalized to the distance between marks, the technique of Jackson et al. [1980] correlates benchmark spacing rather than elevation-dependence.

Strange [1981, his Figure 5] employs another technique to argue for slope-dependence that also suffers from the influence of a trend. His plot of cumulative elevation change as a function of cumulative elevation will always produce a highly significant correlation as long as the trend of neither elevation change (dh) nor topography (dH) curve changes sign. Any tilting elevation change profile and sloping topographic profile will correlate and this can be misinterpreted as a mark-to-mark correlation.

Jackson et al. [1980] do not derive their values for the slope of the regression of elevation change onto elevation, or elevation-dependent error, from the plots such as shown in Figure 14a. Instead they remove the trend from both dh and dH curves by separately fitting each to a 3rd- or 4th-order polynomial. They correlate the residuals to find e_{net}. Implicit in this technique is the assumption that the long wavelength or large scale contribution to the elevation change profile represents tectonic signal, and that the residual high frequency or mark-to-mark signal can be isolated for elevation-dependent correlation. This may not be valid: Abundant evidence from both the field and laboratory suggests that faulting takes place on all spatial scales. Deformation may occur on the scale of the benchmark spacing or over the entire level route. There can be no ideal order poly-

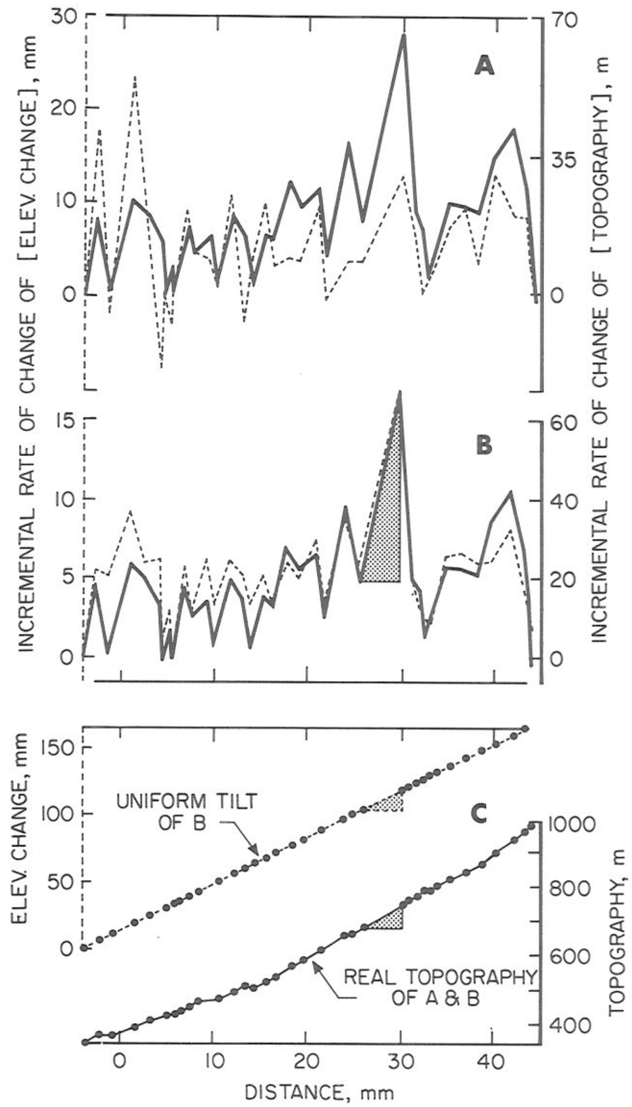


Figure 14. Correlation plot after Jackson et al. [1980] of the 1965-64 surveys of the route from Saugus to Palmdale (shown in Figure 2b). A: correlation of observed elevation change from resurvey, with topography. B: correlation of a uniform tilt with same endpoints and BM-spacing as observed elevation change, with topography. C: the segments correlated in B are shown. Note that when BM's are spaced far apart, a large signal is produced (stippled) since both curves have a positive trend.

nomial. Even if such an a priori segregation of long and short wavelength signals can be justified, residual fitting still suffers from other problems. Since benchmark spacing is never uniform, a sampling bias will persist in the minimization of residuals when fitting the curve. Regions of closely spaced marks will be fit better than those regions where marks are spaced farther apart, and a misfit curve introduces a trend. If the curve fit to the elevation change has too low an order, the trend will not be removed over segments of the profile, and benchmark spacing effects can then dominate. As the order of the polynomial increases, the magnitude of the residuals must drop, and the signal to be correlated consequently diminishes.

Subsidence caused by groundwater withdrawal. Reilinger [1980] argues that the relative subsidence of Saugus with respect to bench marks to the north and south during 1953-1964 (Figure 4a, 0-15 km) is best explained by compaction of the Saugus aquifer, which has a maximum saturated or effective thickness of 600 m. From 1945 through 1967, groundwater withdrawal greatly exceeded recharge. However, records for the two deep wells that tap the Saugus aquifer do not display the long-term head decline that occurred in the overlying alluvial aquifer. Rather, from 1953 to 1966, 90% of the water was pumped from the unconfined alluvial aquifer, which has a maximum saturated thickness of 60 m [Robson, 1972, page 39 and Table 7]. The 0- to 15-km section of the Ridge route (Figures 4a-8a) traverses the alluvial aquifer. It is about thirty times more permeable than the Saugus aquifer, because it is composed of poorly bedded unconsolidated gravel, sand, and silt. To estimate the maximum subsidence within the alluvial aquifer caused by the increase in effective stress, or the stress borne by the aquifer matrix, during the period of pumping, the aquifer compressibility must be measured or modeled. From the hysterisis loops of in situ vertical well extensometers, compressibilities of eight San Joaquin Valley, California, confined aquifers have been measured by Poland et al. [1975], yielding a mean value of $(5 \pm 2.0) \times 10^{-2}$ Nm^{-2}. Because the alluvial aquifer is unconfined and has a coarse grained skeleton, its matrix bears a greater load than the aquifers measured by Poland et al.; pore pressure and seepage stress are lower than in a fine-grained aquifer, resulting in higher permeability and transmissibility, and lower compressibility. Thus these values provide an extreme upper limit. Using the maximum aquifer thickness (60 m) and the portion of the roughly linear 1945-1964 water table decline that occurred between the 1953 and 1964 surveys at Saugus (14 m), differential subsidence of 9 ± 3 mm results. A compressibility of an order of magnitude higher would be required to account for the observed 90 mm differential elevation change between Saugus and bench marks 15 km north or south that Reilinger [1980] ascribes to water withdrawal.

Consider an alternate and independent approach to estimate subsidence related to groundwater withdrawal: the 1953/55-1926 survey includes eight to ten years of water table decline, or about 10 m. No more than 21 ± 7 mm of differential elevation change is evident between Saugus and BM's 15 km to the south, and 12 ± 6 mm with respect to BM's 10-20 km to the north during this period, although 40% of the water was pumped from 1945-1953/55 [Robson, 1972, his Figure 12]. Although pumping continued to 1968 at reduced rates, no more than 10 ± 7 mm of differential subsidence at Saugus can be seen during the 1965-64 and 1968-65 resurveys (Figures 5a and 6a). A maximum estimate of subsidence of 48 ± 20 mm can be obtained by extrapolating the 1945-63 23-m head decline [Robson, 1972, plates 5 and 7] to 30 m through 1968.

Conclusions

Significant correlations between elevation change and elevation have been identified for 65% of the leveling route segments in steep terrain that are suitable for regression. The mean correlation of $(0.3 \pm 2.3) \times 10^{-5} \times dH$ does not change with time; the correlations are as abundant in the 1950's as in the 1970's despite changes in leveling procedure and calibration during this period. The correlations can be removed by a straightforward adjustment. An 100 km long segment through the southern California Transverse Ranges that has been releveled six times shows 165 ± 9 mm of observed uplift at BM G54 near Grapevine with respect to Saugus (J52), and yields 149 ± 18 mm after removal of level rod-related slope-dependent errors between 1953 and 1968. Because sight lengths on this route must be short and were almost the same for each resurvey, differential refraction should have no significant effect on these values of elevation change.

Subsidence caused by groundwater withdrawal from an alluvial aquifer beneath Saugus from 1945-1968 can be approximated by the product of maximum values of the aquifer thickness (60 m) head decline (30 m), and compressibility $(5 \pm 2 \times 10^{-2}\ Nm^{-2})$. This predicts an upper limit of 48 mm of non-tectonic subsidence, leaving a minimum of 100 ± 18 mm of tectonic displacement at G54 during 1953-68.

The 1100 km of resurveys subjected to the test for slope-dependent correlation comprise about 15% of the total population of levels that define the southern California uplift of Castle [1978]. If this sample is representative, the observed elevation change for southern California surveys should be accurate to within $\pm 4.6 \times 10^{-5}$ x the topographic relief, or about 50 mm for most surveys, and 90 mm for 95% of the surveys. The magnitude of the errors is considerably smaller than that of the observed uplift, and about one-fifth the magnitude claimed by Jackson et al. [1980]. Therefore uplift in southern California cannot be the product of any linear slope-dependent error.

A number of tests confirm that the linear slope-dependent errors are related to leveling rods. Fifteen correlations differ significantly where rods are changed, although the spatial resolution of the correlation is precise only to within about 5 km of the rod change. Errors can be assigned to specific rod pairs where multiple lap sections are available, which precludes location- or time-dependent correlations for those cases. Significant changes in correlation without change in rod have not been located, although in some instances these may be masked by frequent rod changes. No elevation-dependent correlations can be found where the same rod was used in both surveys of a route. Finally, circuit misclosures are within the expected mean rod error when many rod changes take place, further substantiating that the errors do not accumulate over distances as large as 300 km.

Several factors contribute to the rod errors, although their relative importance is difficult to assess. Redesign of calibration and encoding practices can reduce current discrepancies, and rod errors larger than about 2×10^{-5} can be identified and removed from some but not all of the historic leveling data.

Appendix: Weighted Linear Regression

The regression equation predicts values of y for given values of x to within an assessable random fluctation. We assume that Y' = a + x, where Y = tilt, Y' = the predicted Y, x = slope, and b, the slope of the regression, is synonomous with e_{net} of eqn. (3), so that

$$b = e_{net} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} , \qquad (8)$$

$$a = \frac{y \Sigma x^2 - \Sigma x \Sigma xy}{n\Sigma x^2 - (\Sigma x)^2} \qquad (9)$$

The y intercept, a, represents the mean uncorrelated tilt, or in other words, the residual tilt after removal of e_{net}. The population regression line is estimated by the method of least squares; the sum of the squares of the deviations in Y are minimized by the regression line. The standard deviation of the slope (here, the standard deviation of e_{net}), S_b, is found by differentiating the equation for slope with respect to Y.

$$S_b = \left(\frac{(Y - bx - a)^2}{n\Sigma x^2 - (\Sigma x)^2}\right)^{1/2} \qquad (10)$$

The sample correlation coefficient [from Crow et al., 1960] is

$$r = b\frac{\sigma_x}{\sigma_y} = \frac{n\Sigma xy - \Sigma x\Sigma y}{([n\Sigma x^2 - (\Sigma x)^2]\ [n\Sigma y^2 - (\Sigma y)^2])^{\frac{1}{2}}} \qquad (11)$$

The significance of r is checked by an equal tails test of the null hypothesis - that no correlation exists. The test requires a greater value of r as the population decreases in size. No correction is applied for auto-correlation. This favors correlation because values of e_{net} are not strictly independent; each survey is used twice.

<u>The weighted regression</u>. In this analysis it is assumed that Y varies from point to point and that X is known exactly. In fact, from one survey to another, Y varies by a factor of 1-10, whereas X varies by no more than 10^{-4}. First the insertion of weights, w, into the equations, and then the rationale for assigning those weights are presented. To adjust the regression line to give more weight to better data, each X_i and Y_i are associated with a W_i inside the summation, and w replaces n. Thus, Y = a + bx becomes $w_i y_i = aw_i + bw_i x_i$, leading to

$$b' = \frac{\Sigma w \Sigma wxy - \Sigma wx\ \Sigma wy}{\Sigma w\Sigma wx^2 - (\Sigma wx)^2},\quad a' = \frac{\Sigma wy\Sigma wx^2 - \Sigma wx\Sigma wxy}{\Sigma w\Sigma wx^2 - (\Sigma wx)^2} \qquad (12)$$

$$\text{and } S_b' = \left(\frac{\Sigma[w(y - b'x - a')^2]}{\Sigma w\Sigma wx^2 - (\Sigma wx)^2}\right)^{\frac{1}{2}} . \qquad (13)$$

Similarly, the weighted regression coefficient of the sample becomes,

$$r_w = \frac{(\Sigma w\Sigma wxy - \Sigma wx\Sigma wy)}{\{[\Sigma w\Sigma wx^2 - (\Sigma wx)^2][\Sigma w\ \Sigma wy^2 - (\Sigma wy)^2]\}^{\frac{1}{2}}} . \qquad (14)$$

To estimate the mean, μ, and the weighted variance, σ_o^2, of the true correlation, for the compilation of all data shown in Figure 10, the variance of correlation coefficients by maximum likelihood is used [Anderson and Bancroft, 1952]. Because the variance of the true correlation is not necessarily equal to that of the sample, we assume that errors in measuring the correlations, y_i, are Gaussian with a known variance, σ_i^2, determined from the linear regression of each resurvey. The true correlations are also assumed to be taken from a Gaussian population. By maximizing the probability with respect to the mean value, we solve

$$\mu = \frac{\Sigma[y_i/(\sigma_i^2 + \sigma_o^2)]}{\Sigma[1/(\sigma_i^2 + \sigma_o^2]} . \qquad (15)$$

The probability of obtaining the observed sample is maximum when

$$\Sigma \left\{\frac{[(y_i - \mu)^2 - \sigma_i^2 - \sigma_o^2]}{(\sigma_i^2 + \sigma_o^2)^2}\right\} = 0 \qquad (16)$$

is satisfied. Note that eqns. (15) and (16) weight the observations by the reciprocal of their variances. σ_o^2 is first set to a value obtained from the population standard deviation of the weighted mean [which yields $(0.9 \pm 4.6) \times 10^{-5}$] and iterated through (15) and (16) until the equality of (16) holds.

Assignment of weights. We assume a Gaussian distribution with known variance, and adopt the relationship between w and the variance [Bacon, 1953], where

$$w_i = 1/\sigma_i^2$$

$$\text{letting } w_i \sigma_{y_i}^2 = w_2 \sigma_{y_2}^2 = \dots w_n \sigma_n^2 = \sigma^2 ,$$

where the w's are the ratios of the variance of each point to some convenience variance taken as the standard. Therefore an observation made where the variance is σ/w is worth w observations in a region where the variance is σ^2. This weighting scheme is employed for both calculation of the population variance in Figure 10 [eqns. (15) and (16)] and for consideration of random errors in the regression [eqns. (12) and (14)].

Because tilts established over longer distances can be more accurately measured, they become a more important tool for testing the dependence of tilt on slope. In geodetic leveling, there is a standard error associated with each measurement of height (at time, t_o, and t_1), where if y = dH,

$$\text{tilt, } Y = \frac{y_{t_1} - y_{t_0}}{dx},$$

$$\text{and } S_y = [(\delta y/\delta t_1)^2 + (\delta y/\delta t_o)^2 \, S_{t_0}^{\,2}]^{\frac{1}{2}} \qquad (17)$$

Taking the partial derivatives with respect to time,

$$\delta y/\delta t_1 = 1/dx \text{ and } \delta y/\delta t_o = -1/dx, \qquad (18)$$

$$\text{so } S_y = (2/dx)^{\frac{1}{2}}$$

Random leveling errors lead to accuracies of $(\text{distance})^{\frac{1}{2}}$; this formula arises from consideration of the sums of squares [Bomford, 1971]. S_{t_O} and S_{t_1} are set equal to each other, since a constant factor, k, will not affect w. Thus,

$$S_y = [(k/dx)^2 \, dx + (k/dx)^2 \, dx]^{\frac{1}{2}} = k \, (1/dx)^{\frac{1}{2}} .$$

Since $w = 1/S_y^2$, w = dx, the distance between benchmarks.

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