Groundwater Velocity Magnitude in Radionuclide Transport Calculations

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Introduction

Analytical solutions have been developed for many conceptual models of solute transport in groundwater (Bear 1979). Although these models usually rely on assumptions too restrictive for accurate description of actual field situations, they are useful in understanding groundwater transport and in evaluating the relative importance of the subsurface processes affecting transport. In addition, these simple models are often used for generic and screening-type analyses of groundwater contamination problems (Kent et al. 1985). For example, the Nuclear Regulatory Commission assesses potential doses resulting from the disposal of very slightly contaminated material in the ground using analytical solutions for one- and two-dimensional groundwater transport (Codell and Schreiber 1979; Codell et al. 1982; Goode et al. 1986). This note presents a method for determining a "worst-case" groundwater velocity value for two conceptual models of decaying radionuclide transport, resulting in maximum calculated point concentration.

For "conservative" screening-type analysis, hydrogeologic properties are typically not known, and assumed parameter values are selected to result in calculated concentrations which are very unlikely to be exceeded in reality. If this type of "conservative" analysis yields performance measures that meet established criteria, then no further information may be required. Otherwise, further site investigation and more realistic analyses can be performed. Parameter selection demands considerable judgment because of the extreme variability of hydrogeologic characteristics from site to site. Fortunately, "worst-case" values can be chosen for some parameters resulting in theoretically maximum or peak calculated performance measure.

Concentrations at a point in an aquifer down gradient from a radionuclide source are affected by, among other processes, dilution in the flowing groundwater and radioactive decay. The travel time of radionuclides from the source to any location determines the extent of radioactive decay. Thus, a higher velocity value results in less decay and higher concentration. On the other hand, groundwater velocity is often considered proportional to flux or specific discharge: $V = q/n$, where q is the specific discharge through the aquifer and n is porosity. The released source mass is diluted by this through-flow, thus a higher velocity value results in more dilution and reduced concentration. These two effects counteract each other and, for certain conceptual models, a groundwater velocity value can

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be determined which will result in maximum concentration at a specified location for a given set of parameters. To simplify notation, this velocity value is herein called the maximum point concentration velocity (MPCV). Performance measures other than maximum point concentration, such as radionuclide mass flux, could also be used to develop different "worst-case" velocity values.

A meaningful MPCV exists only if flux is considered proportional to velocity. In many site-specific cases, groundwater flux can be estimated from recharge or pumping estimates, or from observed hydraulic gradients. Because the amount of dilution is then fixed, a higher velocity value (corresponding to lower porosity) results in less decay and higher concentration, and the MPCV is infinitely high. However, in this case, a more appropriate velocity can be estimated from flux and porosity estimates.

Likewise, radionuclides with very long half-lives do not decay significantly, irrespective of the travel time or velocity. For this case, peak concentration is calculated with minimum-volume flow rate and infinitely low velocity.

This note presents a method for determining a velocity value (MPCV) that will result in maximum concentration at a specified location for a solute subject to rapid or moderate first-order decay when dilution is considered proportional to velocity. A closed-form approximation is developed for two-dimensional advective-dispersive transport. This method is only appropriate when no site-specific information on groundwater velocity is available.

One-Dimensional Plug-Flow Model

A simple conceptualization of radionuclide transport in groundwater is one-dimensional advection (or plug-flow) with linear equilibrium sorption and first-order radioactive decay. This conceptual model ignores dispersion. Initially, concentration is zero for all locations (x). At time $t = 0$, radionuclide mass (measured as activity in curies) is injected at a constant rate M (Ci/T/L) per unit width into the aquifer at $x = 0$. Ahead of the advected front, which is retarded due to sorption, concentration remains zero, because dispersion is ignored. At and behind the advected front, the concentration is:

$$C(t) = C_0 \exp(-\lambda t) \quad \ldots \quad (1)$$

or, substituting $C_0 = M/nbV$ and $t = xR_d/V$,

$$C(x) = \frac{M}{nbV} \exp\left(\frac{-x\lambda R_d}{V}\right) \qquad \text{for } x \leqq \frac{Vt}{R_d} \quad \ldots \quad (2)$$

where C (in Ci/L^3) = radionuclide concentration, in curies (Ci) per unit volume of water; V (in L/T) = uniform groundwater velocity in the x direction; b (L) = aquifer's saturated thickness; n (L^3/L^3) = porosity; R_d (−) = retardation coefficient; and λ (T^{-1}) = radioactive decay rate. The coefficient R_d accounts for sorption of the radionuclide by the aquifer medium and the radionuclides move with an apparent velocity V/R_d (see, e.g., Bear [1979]).

For this case, the sensitivity of Eq. 2 with respect to V is:

TABLE 1. Parameters for Example Problems

Parameter (1)	Symbol (2)	Value (3)
Porosity	n	0.1
Thickness	b	1 m
Source strength	M	1 Ci/yr/m
Retardation	R_d	10
Decay rate	λ	0.021 yr^{-1}
Half-life	—	33 yr

TABLE 2. Calculated Concentrations for One-Dimensional Plug-Flow Model[a]

V (m/yr) (1)	C (x = 100 m) (Ci/m^3) (2)	C (x = 1,000 m) (Ci/m^3) (3)
1	7.58E-9	6.28E-91
2.1	2.16E-5	1.77E-43
10	0.122	7.58E-10
21	0.175	2.16E-5
100	0.081	1.22E-2
210	0.043	1.75E-2
1,000	9.79E-3	8.11E-3
2,100	4.71E-3	4.31E-3

[a] V_1 (x = 100) = $x\lambda R_d$ = 21 m/yr; V_1(x = 1,000) = 210 m/yr.

$$\frac{\partial C}{\partial V} = \left(\frac{x\lambda R_d}{V^2} - \frac{1}{V}\right)C \quad (3)$$

At a local maximum of C, this derivative equals zero. For a nontrivial solution $C \neq 0$ thus, setting Eq. 3 equal to zero and dividing by C yields:

$$V_1 = x\lambda R_d \quad (4)$$

where V_1 = the MPCV for the one-dimensional plug-flow model.

This MPCV depends on the distance from the source at which concentrations are estimated (i.e., the receptor location), the decay rate, and the retardation coefficient. The parameters for an example problem are shown in Table 1. Table 2 illustrates the variation of peak concentration for several velocities at two locations. For this case, an order-of-magnitude underestimate of MPCV results in much lower concentrations than an order-of-magnitude overestimate.

Two-Dimensional Advection-Dispersion Model

A common conceptual model of groundwater transport includes dispersion, or spreading of the radionuclide, both along the flow path (longitudinal) and perpendicular to the flow path (transverse). When dispersion is considered, the MPCV is not equal to Eq. 4 because longitudinal dispersion essentially decreases the travel time of the peak, moving some radionuclides faster than the average.

Wilson and Miller (1978, 1979) developed an approximate solution for the steady-state concentration plume in a uniform flow field for a point source with a constant injection rate M' (in Ci/T). This solution considers both longitudinal and transverse dispersion. The governing equation for this conceptual model is:

$$R_d \frac{\partial C}{\partial t} = \alpha_x V \frac{\partial^2 C}{\partial x^2} + \alpha_y V \frac{\partial^2 C}{\partial y^2} - V \frac{\partial C}{\partial x} - R_d \lambda C \quad \ldots\ldots (5)$$

in which α_x and $\alpha_y(L)$ = the longitudinal and transverse dispersivities, respectively. For $r/B > 1$, an approximate steady-state solution is developed by Wilson and Miller (1978):

$$\bar{C}(x, y) = \frac{M' \exp\left(\frac{x - r}{B}\right)}{nbV(8\pi\alpha_x\alpha_y)^{1/2}\left(\frac{r}{B}\right)^{1/2}} \quad \ldots\ldots (6a)$$

where

$$r = \left[\xi\left(x^2 + \frac{\alpha_x}{\alpha_y} y^2\right)\right]^{1/2} \quad \ldots\ldots (6b)$$

$$B = 2\alpha_x \quad \ldots\ldots (6c)$$

$$\xi = 1 + \frac{2B\lambda R_d}{V} \quad \ldots\ldots (6d)$$

The sensitivity of the centerline ($y = 0$) steady-state concentration to velocity is:

$$\frac{\partial \bar{C}}{\partial V} = \left\{-\frac{1}{V}\left[1 - \frac{x\lambda R_d}{\xi^{1/2} V} - \frac{x\lambda R_d}{2\xi^{1/2} V\left(\xi^{1/2}\frac{x}{B}\right)}\right]\right\}\bar{C} \quad \ldots\ldots (7)$$

Wilson and Dettinger (1979) derived $\partial C/\partial V$ for an exact transient solution to Eq. 5 and tabulated the resulting infinite "sensitivity" integral, although they did not include the effects of velocity change on dispersion. Setting Eq. 7 equal to zero and dividing by $\bar{C}$, the MPCV for the two-dimensional advection-dispersion model, V_2, is:

$$V_2 = \frac{x\lambda R_d}{\left(1 + \frac{2B\lambda R_d}{V_2}\right)^{1/2}}\left[1 + B(2x)^{-1}\left(1 + \frac{2B\lambda R_d}{V_2}\right)^{-1/2}\right] \quad \ldots\ldots (8)$$

This equation must be solved iteratively for V_2. Alternatively, an exact derivative similar to that derived by Wilson and Dettinger (1979) could be derived and numerically minimized to yield MPCV.

Substituting $V_1 = x\lambda R_d$, the one-dimensional plug-flow MPCV velocity, Eq. 8 can be written:

$$\frac{V_2}{V_1} = \left(1 + \frac{2BV_1}{xV_2}\right)^{-1/2}\left[1 + \frac{B}{2x}\left(1 + \frac{2BV_1}{xV_2}\right)^{-1/2}\right] \quad \ldots\ldots (9)$$

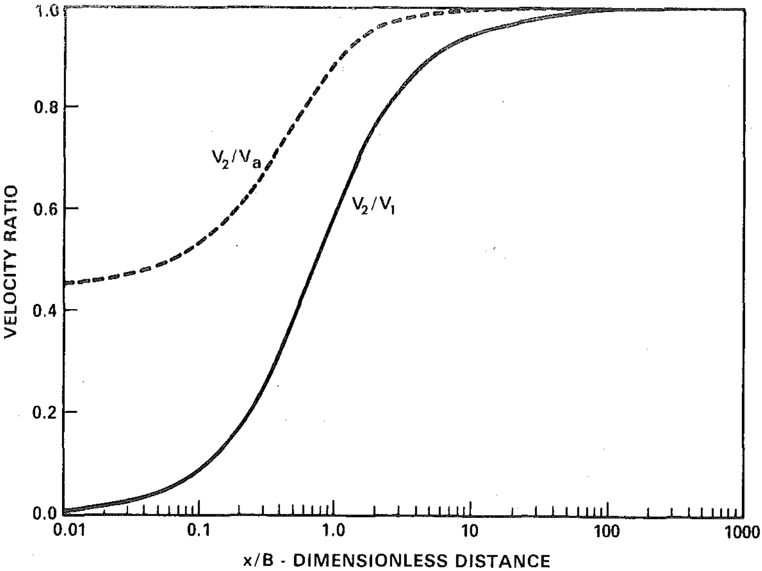

FIG. 1. Ratio of Two-Dimensional Advection-Dispersion MPCV (V_2) to Approximate MPCV (V_a) and to One-Dimensional Plug-Flow MPCV (V_1)

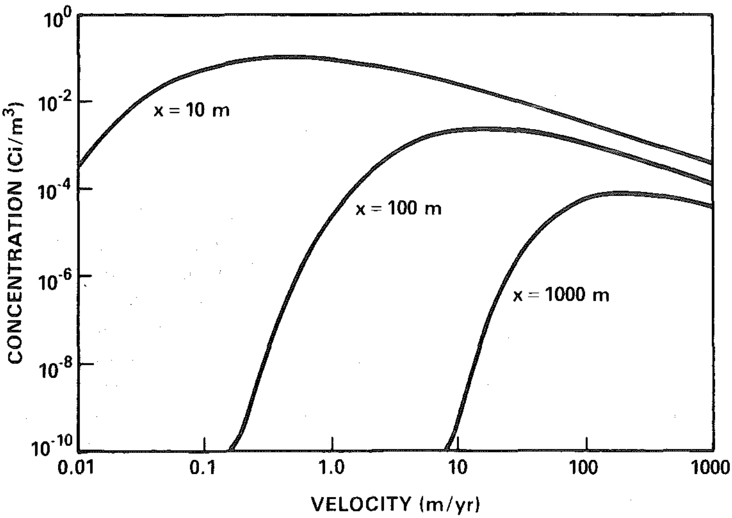

FIG. 2. Centerline Peak Concentration in Two-Dimensional Plume at Steady State versus Velocity for Three Distances Down Gradient

The ratio of MPCV for the two-dimensional advection-dispersion model to the MPCV for the one-dimensional plug-flow model is a function of x/B only (Fig. 1). This term $x/B = x/2\alpha_x$ is one-half the dimensionless distance down gradient from the source relative to the longitudinal dispersivity.

The form of the one-dimensional MPCV with respect to the one-dimensional analytical solution suggests an approximation (V_a) to the MPCV for the two-dimensional advection-dispersion model:

$$V_a = \frac{2B\lambda R_d}{2\left(\frac{B}{x}\right) + \left(\frac{B}{x}\right)^2} \qquad (10)$$

For large x/B, the second term in the denominator in Eq. 10 is small, and V_a reduces to V_1 (Eq. 4) in the limit.

Fig. 1 shows the ratio of V_2 (Eq. 7) to V_1 (Eq. 4) and to V_a (Eq. 10) as a function of dimensionless distance from the source. V_a is a better approximation of V_2 than V_1, because it incorporates some of the affect of dispersion. Recalling that $B = 2\alpha_x$, V_a is about 90% of V_2 for $x = 2\alpha_x$, and V_a is greater than 99% of V_2 for $x = 20\alpha_x$.

The variation of steady-state centerline concentration for different assumed groundwater velocities is shown in Fig. 2. The exact analytical solution (to which Eq. 6 is an approximation, see Wilson and Miller [1978]) is used for a case governed by the parameters in Table 1. In addition, the longitudinal and transverse dispersivities are 20 m and 4 m, respectively. It should be noted from Eq. 8 that MPCV is independent of transverse dispersivity, although the absolute sensitivity (Eq. 7) is a function of α_y through its affect on $\bar{C}$. For the case considered, centerline concentration is more sensitive to velocity for velocities less than MPCV, when decay is most important. The sensitivity is less when velocity is above MPCV and dilution effects become dominant. These characteristics are also shown above in Table 2, for the plug-flow model.

Summary

For certain generic or screening-type models of radionuclide transport in groundwater, a groundwater velocity value (MPCV) can be determined which will result in maximum calculated concentration at a specified location. The MPCV is a function of the distance from the source, the radioactive decay rate, the retardation coefficient, and the longitudinal dispersivity. For the two-dimensional advection-dispersion model considered, the MPCV can be determined by a simple approximation multiplied by the ratio shown in Fig. 1. For many cases, the MPCV for a simple one-dimensional advection model may provide a reasonable estimate of the MPCV for more complex models which can be verified by computing concentrations corresponding to higher and lower velocity values. This method is presented in terms of transport of a decaying radionuclide in groundwater, although it may also be applied to other problems such as surface water transport and transport of organic solutes subject to first-order biodegradation. MPCVs could also be developed for other conceptual models including three-dimensional transport, different source configurations, or additional geochemical processes. This method is only appropriate when no site-specific information on groundwater velocity is available.

Acknowledgment

The contents of this note represent the opinion of the writer and do not necessarily represent the official policy of the U.S. Nuclear Regulatory Commission.

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Plunging and Streaming Flows in Pool and Weir Fishways

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Introduction

A pool-and-weir fishway consists of a number of pools formed by a series of weirs. Water flows from the headwater side to the tailwater region

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