Composite Recovery Type Curves in Normalized Time from Theis' Exact Solution

by Daniel J. Goode[a]

Abstract

Type curves derived from Theis' exact nonequilibrium well function solution are proposed for graphical estimation of aquifer hydraulic properties, transmissivity (T), and storage coefficient (S), from water-level recovery data after cessation of a constant-rate discharge test. Drawdown (on log scale) is plotted versus the ratio of time since pumping stopped to duration of pumping, a normalized time. Under Theis conditions, individual type curves depend on only the dimensionless pumping duration, which depends in turn on S and radial distance from the pumping well. Type curve matching, in contrast to the Theis procedure for pumping data, is performed by shifting only the drawdown axis; the time axis is fixed because it is a relative or normalized time. The match-point for the drawdown axis is used to compute T, and S is determined from matching the curve shape, which depends on early dimensionless-time data. Multiple well data can be plotted and matched simultaneously (a composite plot), with drawdown at different radial distances matching different curves. The ratio of dimensionless pumping durations for any two matched curves is equal to one over the squared ratio of radial distances. Application to two recovery datasets from the literature confirm the utility of these type curves in normalized time for composite estimation of T and S.

Introduction

Theis (1935) presented the exact solution for nonequilibrium drawdown during constant rate pumping in an infinite, homogeneous, horizontal flow, confined aquifer, and developed the type curve procedure for estimation of transmissivity (T) and storage coefficient (S) from this exact well function solution. He also presented a large dimensionless-time approximation from which a simpler slope matching graphical procedure was later developed (Cooper and Jacob, 1946).

Theis (1935) also presented the exact solution for water-level recovery after cessation of pumping by superposition of continuing drawdown due to the original pumping, and negative drawdown, or recovery, due to injection at the same rate beginning at the cessation of pumping. However, his method for estimation of T from recovery data was based on the large time approximation, rather than the exact well function superposition. Furthermore, this large time approximation is independent of S, and hence S cannot be estimated using this method. Horner (1951) presents the same solution in the context of pressure build-up testing, which has been used extensively in the petroleum industry.

One common method to estimate S from recovery data is to treat "recovery," R, as drawdown during pumping (U.S. Department of the Interior, 1985). R is defined as the drawdown due to pumping extrapolated to an observation time beyond the end of pumping, minus the observed drawdown. [In many references the drawdown after pumping ceases is called "residual" drawdown.] Theoretically, this is exactly the negative drawdown due to an injection starting at the end of pumping; hence this data can be analyzed as a pump test, and T and S can be determined. The main drawback with this method is the extrapolation of pumping drawdown (Ballukraya and Sharma, 1991); it is especially subjective when performed graphically. Nongraphical extrapolation can be performed using the well function (Fenske, 1977), or the large time approximation (Ballukraya and Sharma, 1991), but then T and S are essentially known a priori, or at least estimated from drawdown during both pumping and recovery. Because R depends on the extrapolated drawdown during pumping, estimation of T and S can be significantly in error when well loss or erratic pumping rates affect the drawdown during pumping.

Ramey (1980) similarly presents type curves for drawdown during both the pumping and recovery periods on a single log-log plot based on Theis' (1935) exact solution. Plotting drawdown during the recovery period on the time-since-pumping began scale means, however, that all recovery times are plotted as large times. This may obscure some of the initial features of recovery that occur early relative to total recovery but at large time relative to the time since pumping began. Furthermore, as noted above, this method is limited to cases where the drawdown during the pumping period agrees with that predicted by the Theis equation, which may not be the case, particularly for the pumped well. Mishra and Chachadi (1985) present recovery type curves similar to those of Ramey (1980) for drawdown during and after pumping, accounting for borehole storage in the pumped well. A more versatile method would be independent of the results of the pumping period analysis, particularly for cases where erratic pumping rates or well loss limit the use of pumping period data.

Case et al. (1974) present a method for analysis of recovery data that is independent of the pumping period data and allows direct estimation of S. This method is based on a series approximation to the well function, but retains more terms than the large

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time approximation. T and S can be computed for any two observed drawdowns during the recovery period. For a test with many observations, multiple estimates are generated and can be averaged or treated statistically. Subba Rao and Gokhale (1986) apply Case's method, which they call the two-point method, and present a computer program for the tedious calculations associated with the series approximation. Recently, Banton and Bangoy (1996) present a similar but simpler graphical method using the first three, as opposed to Theis' first two, terms of the series approximation. Although simpler than Case's method, the method of Banton and Bangoy (1996) nonetheless involves three separate plots and requires at least two obvservation wells at different radial distances.

The exact solution for recovery is used in automatic optimization procedures for estimation of T and S by Bardsley et al. (1985), Almeida (1987), and Kashyap et al. (1988). Their methods are based on Theis' (1935) exact well-function solution for the recovery period. Because the optimization can be defined to include all data simultaneously, a composite estimate of aquifer properties using multiple observation wells, in addition to the pumped well, is possible (Kashyap et al., 1988). Almeida (1987) and Kashyap et al. (1988) illustrate their model fits by plotting drawdown versus time since pumping ceased on log-log scales.

In this paper, a graphical recovery type curve procedure based on Theis' (1935) exact well-function solution is proposed; it corresponds to the Theis log-log type curve in that early dimensionless-time response is included and graphical matching of the type curve and observed drawdown yields T and S. The method is conceptually similar to the optimization method of Kashyap et al. (1988) and can be thought of as an application of their approach using graphical instead of numerical fitting. The normalized time framework proposed here is convenient for composite analysis of multiple well data and may also offer advantages in alternative numerical fitting procedures. These type curves may be used in place of Horner (1951) plots for analysis of pressure buildup, particularly for analysis of "interference" or observation well data.

Significant progress has been made in estimation of aquifer properties from water-level recovery in wells where the Theis assumptions are not fully satisfied. Recent work includes both analytical and numerical modeling to account for the effects of borehole storage and other nonideal features (Zdankus, 1974; Fenske, 1977; Hargis, 1979; Rushton and Holt, 1981; Mishra and Chachadi, 1985; Schmitt, 1988; Tripp and Christian, 1989). These effects are not considered in this paper, but some of them may be conveniently treated in the proposed normalized time framework.

The methods proposed here are strictly applicable only if all Theis assumptions are valid, which is arguably never true. For example, most actual aquifers are not fully confined and receive some recharge from over or underlying units. However, as with the Theis procedure for pumping period data, comparison of observed drawdown to the theoretical type curves presented here may be a first step in identifying aquifer properties from recovery data (for example, Neuman, 1975). As appropriate, more complex analytical and numerical models of recovery can be developed for site-specific hydrogeologic conditions. Furthermore, many nonideal features may have little effect on drawdown, particularly at observation wells, such that estimates derived from these methods approximately characterize field-scale aquifer properties. This is illustrated by the second example in this paper, a water-table case, which has been analyzed successfully before using Theis confined-aquifer methods (Dagan, 1967).

Theory

Theis' Analytical Solution

Theis (1935) considers constant-rate pumping from an infinitesimally small well that fully penetrates an infinite, homogeneous, fully confined aquifer. The exact solution for drawdown during pumping is

$$s(r, t) = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-y}}{y} dy = \frac{Q}{4\pi T} W(u) , \tag{1}$$

whereas s [L] is drawdown (initial head minus head as a function of r and t); T $[L^2/T]$ is isotropic transmissivity; S [dimensionless] is storage coefficient; Q $[L^3/T]$ is the constant pumping rate; W(u) is Theis' well function; u [dimensionless] is $r^2S/4Tt$; r [L] is radial distance from the center of the pumping well; and t [T] is time. Defining dimensionless drawdown, s_D, and dimensionless time, t_D, as

$$s_D \equiv \frac{4\pi Ts}{Q} ; \quad t_D \equiv \frac{Tt}{r^2S} = \frac{1}{4u} , \tag{2}$$

the solution during pumping (1) can be written $s_D(t_D) = W(1/4t_D)$.

At large dimensionless time, when $t_D > 25$ ($u < 0.01$), the well function can be closely approximated by a simple logarithmic expression (Theis, 1935; Cooper and Jacob, 1946):

$$s_D(t_D > 25) = 2.3 \log_{10}(2.25\, t_D) . \tag{3}$$

Large dimensionless time depends on both time and radial distance, and will occur much sooner at locations close to the pumped well than at distal points.

Theis (1935) also derived the analytical solution for the water-level recovery after cessation of pumping:

$$s_D(t_D > t_{pD}) = W\left(\frac{1}{4t_D}\right) - W\left(\frac{1}{4(t_D - t_{pD})}\right) = \int_{1/4t_D}^{1/4(t_D - t_{pD})} \frac{e^{-y}}{y} dy \tag{4}$$

where t_{pD} is the dimensionless duration of pumping (Ramey, 1980), defined by substituting the pumping duration t_p [T] for t in (2).

$$t_{pD} \equiv \frac{Tt_p}{r^2S} , \tag{5}$$

and this dimensionless pumping duration depends on the radial distance of the observation point.

Theis' (1935) procedure for analyzing the recovery data is based on the large dimensionless-time approximation to (4), appropriate when $(t_D - t_{pD}) > 25$,

$$s_D(t_D > t_{pD} + 25) = 2.3 \log_{10}\left(\frac{t_D}{t_D - t_{pD}}\right) . \tag{6}$$

In dimensional variables, this approximate solution can be written

$$s(t > t_p + 25r^2S/T) = 2.3 \frac{Q}{4\pi T} \log_{10}\left(\frac{t}{t - t_p}\right) . \tag{7}$$

Theis' (1935) method of analysis of recovery data is based on equation (7), which does not depend on either radius r or storage coefficient S. Hence S cannot be estimated.

Normalized Time

A normalized time since pumping ceased is defined as the ratio of time since pumping stopped to the duration or pumping:

$$t_n \equiv \frac{t - t_p}{t_p} = \frac{t_D - t_{pD}}{t_{pD}}, \tag{8}$$

and the exact solution can be written

$$s_D(t_n > 0) = W\left(\frac{1}{4(t_n t_{pD} + t_{pD})}\right) - W\left(\frac{1}{4t_n t_{pD}}\right). \tag{9}$$

Theis' large time approximation becomes

$$s_D\left(t_n > \frac{25}{t_{pD}}\right) = 2.3 \log_{10}\left(\frac{t_n + 1}{t_n}\right). \tag{10}$$

Figure 1 shows the dimensionless drawdown as a function of normalized time since pumping ceased (t_n), for several values of dimensionless pumping duration (t_{pD}). The log-log plot for illustration of drawdown during only the recovery period has been used by Almeida (1987) and Kashyap et al. (1988), although the normalized time used here is new. The exact solution (9) in the normalized time form depends on r and S through t_{pD}, the dimensionless pumping duration. The limiting upper curve ($t_{pD} \to \infty$) in the figure is the large time approximation of Theis (1935), equation (10). At very large normalized time, the log-log plot of dimensionless drawdown versus normalized time has a slope of -1, which can be observed from the limiting form of (10):

$$\lim_{t_n \to \infty} s_D = \lim_{t_n \to \infty} \ln\left(1 + \frac{1}{t_n}\right) = \frac{1}{t_n}. \tag{11}$$

This is the large time "line-source" slug-test solution of Ferris and Knowles (1963), to which the finite-well Cooper-Bredehoeft-Papadopulos (1967) slug-test solution converges at very large time. The recovery solution here is viewed in the slug-test framework by recognizing Qt_p as analogous to the slug volume.

Type Curve Method for Transmissivity and Storage Coefficient

The exact dimensionless solution of water-level recovery can be used in a type curve matching procedure to estimate transmissivity T and, in some cases, storage coefficient S. Traditional Theis type curve matching for pumping period data is conducted by overlaying the dimensionless log drawdown versus log time plot and a same scale plot of the field data and shifting both axes until a good match is obtained. In the present case, the normalized time axis is fixed and does not shift. Hence, this axis can be plotted in any format; the logarithmic scale is chosen for the presentation here.

A field data plot can be prepared using the same scales as Figure 1 and the appropriate logarithmic ranges of drawdown based on observations. The normalized time axis is the time since

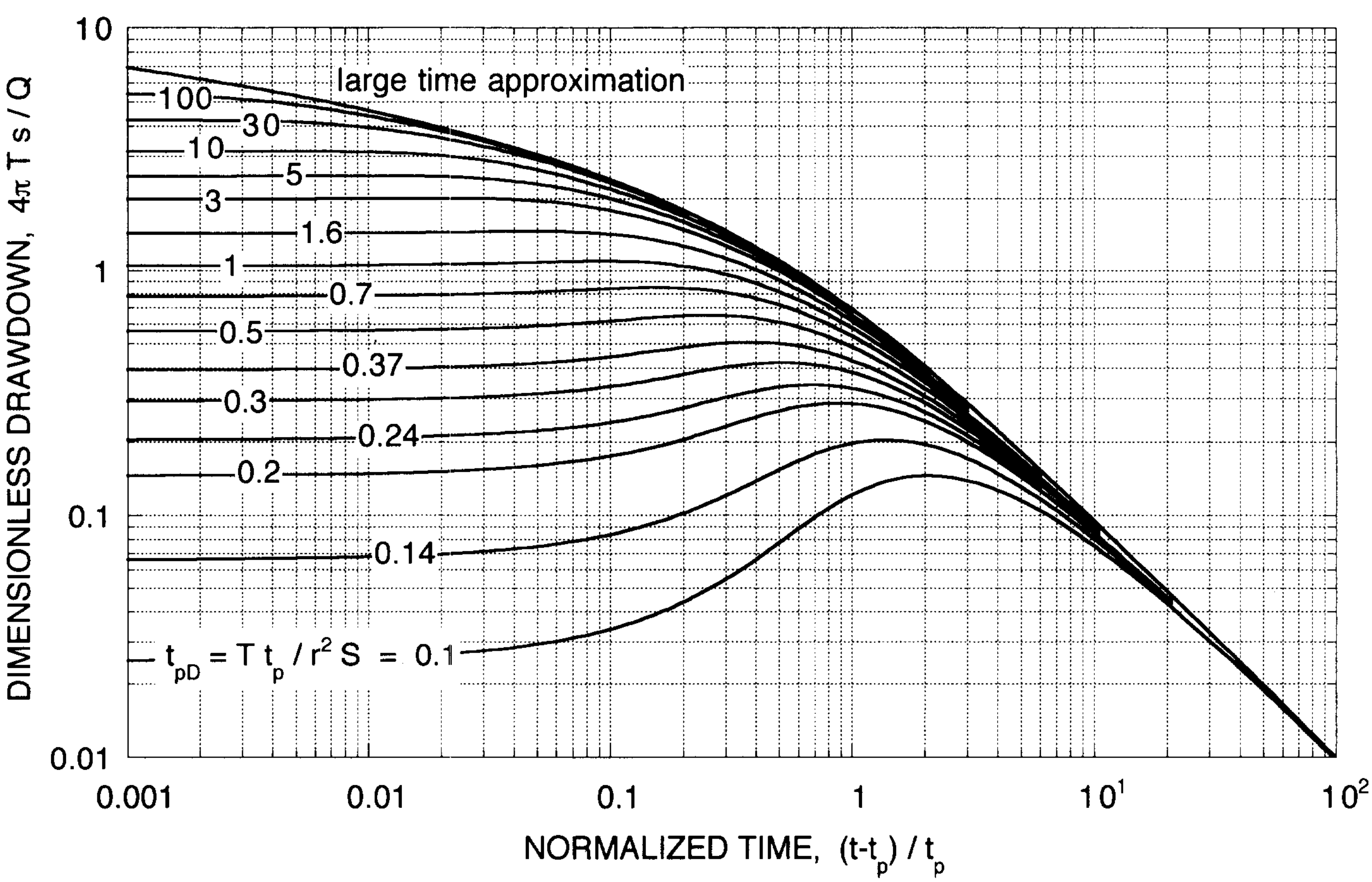

Fig. 1. Log-log composite recovery type curves in normalized time for water-level recovery after cessation of constant-discharge pumping under Theis conditions. Dimensionless drawdown, $s_D = 4\pi Ts/Q$ is plotted versus normalized time, the time since pumping stopped divided by pumping duration, for several values of dimensionless pumping duration, $t_{pD} = Tt_p/r^2S$.

pumping stopped divided by the duration of pumping. The plots are overlaid with identical normalized time scales. The data plot is shifted vertically until a suitable match between the observed and theoretical data and curve shape is achieved. As with the standard Theis procedure for drawdown during pumping, a match point is chosen, in this case for drawdown (s') and dimensionless drawdown (s'_D) only. The transmissivity is computed from

$$T = \frac{s'_D}{s'} \frac{Q}{4\pi} . \tag{12}$$

The storage coefficient is determined from the particular curve matched, associated with a dimensionless pumping duration t'_{pD}. The storage coefficient is then computed from

$$S = \frac{t_p}{t'_{pD}} \frac{T}{r^2} . \tag{13}$$

where r is the radius of the observation well, and t_p is the actual pumping duration. Because the curves for different values of t_{pD} are somewhat similar, the estimation of S is likely to be less certain than T. To facilitate curve selection, additional curves between those shown in Figure 1 can be plotted for specific values of t_{pD} using available well function solutions (a computer program to calculate solutions for any value of t_{pD} is available from the author).

The dimensionless drawdown and observed data can easily be plotted together by multiplying dimensionless drawdown by a scale factor (equivalent to s'/s'_D [L]). This procedure can be implemented in a computer plotting or spreadsheet program. The drawdown and scaled dimensionless drawdown are then plotted in any useful axis format using real length units. When a satisfactory match is achieved between drawdown and the scaled dimensionless drawdown, the optimum scale factor is inserted in equation (12) to compute T. Furthermore, S is determined from the dimensionless pumping duration of the best-match curve using (13).

The existing method of extrapolating drawdown beyond the end of pumping and analyzing computed "recovery" R using the Theis equation yields both T and S (U.S. Department of the Interior, 1985). If both pumping and recovery period drawdown are governed by the Theis model, the proposed method yields theoretically identical results to the recovery-as-drawdown method. However, the proposed method is simpler because it avoids the extrapolation step; measured drawdown during the recovery period is used directly.

When recovery data from multiple wells are available, each well can be analyzed separately and the results treated as multiple estimates. If the assumptions are realistic, including isotropy, the estimated values should have a small range. In anisotropic aquifers, available procedures can be used to estimate the transmissivity tensor from separate analyses of observation well data at different orientations (for example, Maslia, 1994).

A composite procedure is proposed for isotropic aquifers. In this case, data from all wells can be plotted on a single graph and all data matched simultaneously. A single match point for the vertical drawdown axis gives a composite estimate of T. Data from wells at different radial distances fall on different curves because t_{pD} depends on r. However, the matched curve t'_{pD} values are explicitly related to each other. The relation between the matched curves t'_{pD} values for two wells at different radii r_A and r_B is

$$\frac{t'_{pD}(r_A)}{t'_{pD}(r_B)} = \frac{\dfrac{Tt_p}{r_A^2 S}}{\dfrac{Tt_p}{r_B^2 S}} = \left(\frac{r_B}{r_A}\right)^2 . \tag{14}$$

If this relation is enforced, then a composite estimate of S can be obtained from any of the curves using (13). A practical scheme for this procedure is to match data from one well first, compute dimensionless data for the other wells using dimensionless pumping durations computed from (14) and test the quality of the match between all generated type curves and all observations in a single data plot. Heterogeneity, anisotropy, partial penetration, and water-table conditions, among other factors, can cause poor matches in the composite plot.

Examples

Pichaco Dam (U.S. Department of the Interior, 1985)

Recovery in wells at Pichaco Dam (U.S. Department of the Interior, 1985) is analyzed by the proposed method to illustrate the procedure for estimation of T and S from measurements at the pumped well and an observation well. Unfortunately, the reference gives little information about the hydrogeology at the site. The well was pumped for 800 minutes at a rate of 0.077 m^3/s. Using Theis' recovery method applied to both the pumping well and observation well recovery, T was estimated as 0.0491 and 0.0497 m^2/s, respectively (U.S. Department of the Interior, 1985). Recovery at the observation well was analyzed as a pumping test to estimate $S = 0.07$. As summarized in the Introduction, the storage coefficient was estimated from the data during the recovery period, but only after extrapolation of the pumping period drawdown. The alternative method proposed here does not require this extrapolation and S is estimated directly from observed drawdown during the recovery period. Ballukraya and Sharma (1991) also analyzed these data using a theoretical extrapolation method and estimated the storage coefficient as 0.06.

Figure 2 is an overlay of the type curve from Figure 1 and a log-log plot of drawdown versus normalized time at the pumped well ($r \approx 0$; the radius of the pumped well is not given in the reference) and from an observation well at $r = 30.5$ m. A match point is chosen where $s' = 1$ m and $s'_D = 7.7$. The transmissivity is computed from (12) as $T = 0.0472$ m^2/s, slightly less than previous estimates. This is a composite estimate of T from drawdown at both wells.

As predicted, after only a short period of time, drawdown at the pumped well and nearby observation wells is essentially the same, and hence the drawdown is independent of r and S. consistent with Theis' large time approximation. However, matching the early dimensionless-time data from the observation well to a particular curve, corresponding to a specific value of dimensionless pumping duration, allows S to be estimated. Figure 3 is a log-log plot of the drawdown at the observation well at $r = 30.5$ m and the dimensionless drawdown corresponding to $t_{pD} = 35$. Using (13), the storage coefficient is estimated as 0.07. Also shown in Figure 3 is the maximum drawdown observed during the pumping period which agrees well with the type curve prediction at very small normalized time for $t_{pD} = 35$.

The maximum drawdown in the pumped well after 800 minutes of pumping (3.81 m) shown in Figure 2 is higher than that predicted (2.65 m) using the Theis equation with the esti-

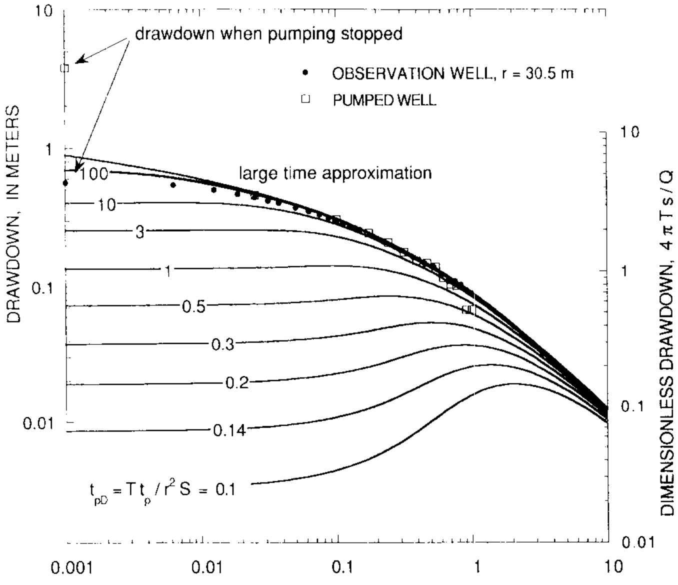

Fig. 2. Overlay of recovery type curves and observed drawdown during recovery at the pumped well and at an observation well at r = 30.5 m at Pichaco Dam (U.S. Department of the Interior, 1985). Match point of s′ = 1.0 m and s'_D = 7.7 gives transmissivity of T = 0.0472 m^2/s.

mated T and S and an assumed well radius of 0.01 m or 1 cm. This exaggerated drawdown may reflect well loss, and illustrates typical difficulties in using pumped well data, particularly at early time. The actual well radius, which was not reported, is surely larger than 1 cm, and use of a larger well radius would yield even smaller predicted drawdown. Furthermore, the observed drawdown immediately after the pump was stopped was actually negative (U.S. Department of the Interior, 1985), probably due to water from the discharge pipe falling back into the borehole. Despite these typical but nonideal features, drawdown during the recovery period agrees well with the theoretical prediction soon after pumping is stopped. Well loss effects disappear rapidly with velocity decreases following cessation of pumping.

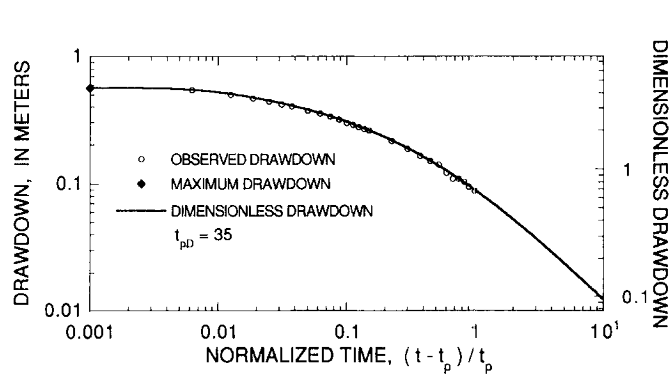

Fig. 3. Water-level recovery at an observation well 30.5 m from the pumped well at Pichaco Dam (U.S. Department of the Interior, 1985) plotted with scaled theoretical drawdown during recovery for dimensionless pumping duration t_{pD} = 35, corresponding to storage coefficient S = 0.07 and transmissivity T = 0.0472 m^2/s. Maximum drawdown observed when pumping stopped is also shown.

Grand Island, Nebraska (Wenzel, 1942)

Wenzel (1942) reports an extensive drawdown dataset for a pumping test in the Platte River Valley near Grand Island, Nebraska. The pumped well partially penetrates the water-table aquifer, which is approximately 30 m thick. The 0.61 m diameter well was pumped at a rate of 0.034 m^3/s for 48 hours. Drawdown was observed in the pumped well only during the recovery period. Drawdown during both the pumping and recovery periods was measured at 83 shallow piezometers (diameters of 2.5 to 7.5 cm) ranging in radial distance from about 1 m to over 360 m. Adjusting drawdown during the pumping period to account for partial penetration effects, Jacob (1963a) estimated T = 0.026 m^2/s, significantly higher than Wenzel's estimate (0.016 m^2/s) which did not account for partial penetration. Wenzel estimated the specific yield as 0.217 from analysis of pumping drawdown data and as 0.201 from volume-drained calculations. Dagan (1967) analyzed the drawdown during pumping using a model accounting for vertical flow and storage from the water table alone. He estimated T = 0.049 m^2/s and the ratio of vertical to horizontal hydraulic conductivity as 0.13. From his model, Dagan (1967) suggests that the Theis model, which ignores partial penetration and vertical flow, is a good approximation for $r > 1.2\,B\,(K_h/K_v)^{1/2}$, where B [L] is the saturated thickness, and K_h and K_v [L/T] are the horizontal and vertical hydraulic conductivities, respectively. Using his estimates, the Theis model should be adequate for r > about 100 m. Dagan used the Theis method to estimate average T = 0.052 m^2/s and average S = 0.202 from drawdown at six piezometers at radial distances greater than about 180 m.

Wenzel used Theis' large time procedure for analysis of the recovery period drawdown at the pumped well and estimated T = 0.017 m^2/s. This procedure does not provide an estimate of S. Subba Rao and Gokhale (1986) estimated S as 0.1287 from analysis of pumped well recovery. Jacob (1963b) used Theis' method with a nonconstant storage coefficient to analyze recovery in the pumped well and found a difference between pumping and recovery period S of 22 percent.

Drawdown at the pumped well exhibits some deviation from the type curves presented here, probably because the proposed method does not account for borehole storage or partial penetration in this water-table aquifer. Drawdown decreases too rapidly at late time. The effect of borehole storage may be accommodated in the proposed normalized time type curve method using available solutions (Papadopulos and Cooper, 1967; Moench, 1984). Matching drawdown during recovery in the pumped well to the Theis type curve yields an estimated T of 0.019 m^2/s, between Wenzel's and Jacob's results.

The proposed composite analysis is used to estimate both T and S from recovery at four piezometers. Figure 4 shows the overlay of the type curves of Figure 1 and the recovery period drawdown at the pumped well and four piezometers, well 48 at r = 87 m, well 19 at r = 129.4 m; well 10 at r = 230 m, and well 70 at r = 371 m. The composite match (Figure 4) is manually weighted towards the piezometers, ignoring the pumped well drawdown. This reduces the nonideal effects of partial penetration and borehole storage in the pumped well. The drawdown at 230 m matches the type curve for dimensionless pumping duration of about 0.5.

The composite match using specific type curves corresponding to the actual piezometer radial distances is shown in Figure 5. Specific type curves for the other three observation wells are

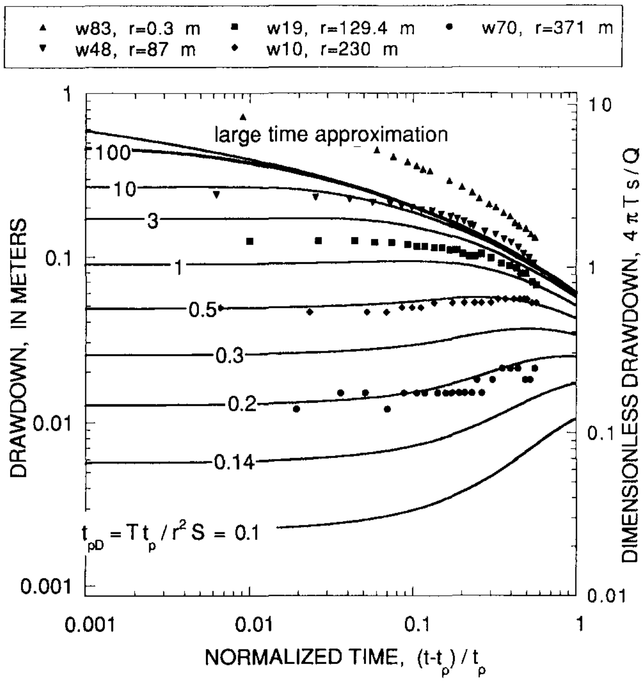

Fig. 4. Overlay of composite recovery type curves and observed drawdown during recovery at the pumped well and at four observation wells near Grand Island, Nebraska (Wenzel, 1942). Match point of $s' = 0.1$ m and $s'_D = 1.18$ gives transmissivity of $T = 0.032$ m^2/s.

generated from the match at r = 230 m using the prescribed relation between dimensionless pumping duration and radial distance (14). The fit is good for three of the piezometers, but the drawdown at r = 87 m is somewhat higher than predicted. Based on Dagan's (1967) estimate, the three piezometers with a good fit

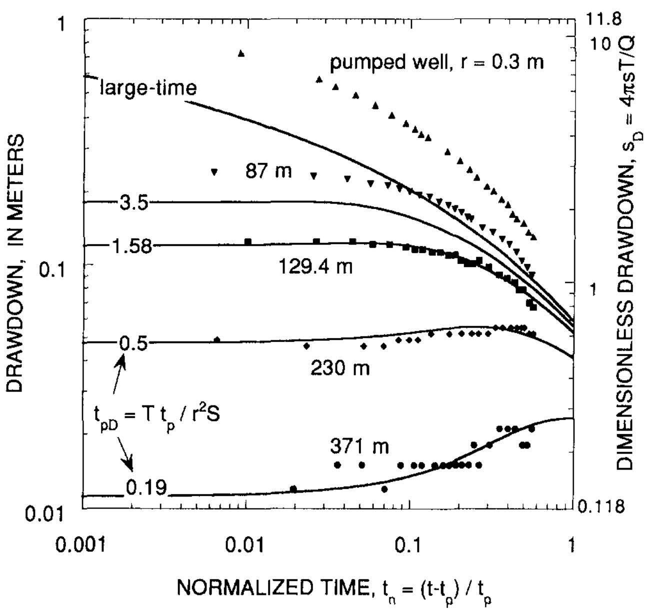

Fig. 5. Composite plot of water-level recovery at the pumped well and at four observation wells near Grand Island, Nebraska (Wenzel, 1942) and scaled theoretical drawdown during recovery for specific values of dimensionless pumping duration corresponding to storage coefficient of S = 0.209 and transmissivity T = 0.032 m^2/s. The dimensionless pumping durations shown satisfy constraint equation (14): $t_{pD}(A)/t_{pD}(B) = (r_B/r_A)^2$.

are sufficiently far from the pumped well for partial penetration and vertical flow effects to be negligible. All of the drawdown data seem to indicate more rapid recovery at late time than predicted by the theory, perhaps due to recharge from the unsaturated zone. The transmissivity estimated from the composite fit, weighted towards the observations at larger r, is T = 0.032 m^2/s, about 23 percent higher than Jacob's (1963a) estimate and about 62 percent of Dagan's (1967) estimate. From (13), the storage coefficient is estimated as 0.209, in agreement with Wenzel's and Dagan's (large radial distance) estimates from pumping period drawdown.

Summary

A new graphical composite type curve method is proposed to estimate transmissivity and storage coefficient from water-level recovery data. The normalized time type curves are advantageous because all type curves converge to the large dimensionless-time approximation, the differences between curves as a function of storage coefficient are accentuated, and the time axis is not shifted during curve matching. Two example applications highlight the estimation of S from early dimensionless-time recovery data, composite curve matching for multiple wells, and limitations in using pumped well recovery without accounting for borehole storage, partial penetration, well loss, and other nonideal features. Theis' large time method cannot be used for the wells matched in the Grand Island example because all observations are at early dimensionless time at these large radial distances. Although real settings will not fully satisfy Theis' assumptions, application of these simple methods can be a useful first step in characterizing aquifer properties from recovery data. The Grand Island water-table example here illustrates application of these methods to drawdown from wells sufficiently far from the pumped well that partial penetration and water-table effects can be ignored. Compared to the very recent method of Banton and Bangoy (1996), the methods proposed here are simpler and require only one observation well for estimation of S. If additional wells are available, the methods proposed here allow a composite estimation of T and S using drawdown at all wells together, provided the Theis model is a reasonable approximation.

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