WATER RESOURCES RESEARCH, VOL. 32, NO. 2, PAGES 289–296, FEBRUARY 1996

Direct simulation of groundwater age

Daniel J. Goode
U.S. Geological Survey, Malvern, Pennsylvania
Princeton University, Princeton, New Jersey

Abstract. A new method is proposed to simulate groundwater age directly, by use of an advection-dispersion transport equation with a distributed zero-order source of unit (1) strength, corresponding to the rate of aging. The dependent variable in the governing equation is the mean age, a mass-weighted average age. The governing equation is derived from residence-time-distribution concepts for the case of steady flow. For the more general case of transient flow, a transient governing equation for age is derived from mass-conservation principles applied to conceptual "age mass." The age mass is the product of the water mass and its age, and age mass is assumed to be conserved during mixing. Boundary conditions include zero age mass flux across all noflow and inflow boundaries and no age mass dispersive flux across outflow boundaries. For transient-flow conditions, the initial distribution of age must be known. The solution of the governing transport equation yields the spatial distribution of the mean groundwater age and includes diffusion, dispersion, mixing, and exchange processes that typically are considered only through tracer-specific solute transport simulation. Traditional methods have relied on advective transport to predict point values of groundwater travel time and age. The proposed method retains the simplicity and tracer-independence of advection-only models, but incorporates the effects of dispersion and mixing on volume-averaged age. Example simulations of age in two idealized regional aquifer systems, one homogeneous and the other layered, demonstrate the agreement between the proposed method and traditional particle-tracking approaches and illustrate use of the proposed method to determine the effects of diffusion, dispersion, and mixing on groundwater age.

Introduction

The age of water in a groundwater system is useful for quantitative analysis. Various environmental tracers can be used to estimate the time since recharge for groundwater samples collected from various locations. These age data can be used in turn to constrain the parameters of models of flow and transport. Isotopic information is typically interpreted by using a travel time approach, simulating groundwater movement as "piston" flow. *Reilly et al.* [1994] show that an advective model of groundwater age, simulated by numerical particle tracking, is consistent with the distribution of chlorofluorocarbons and tritium observed in a shallow sand and gravel aquifer. In the advective model the age is determined by the travel time of water, computed from Darcy's law. However, isotope transport in groundwater is often not by advection alone.

Several studies have indicated that estimates of groundwater travel time can be in error if the effects of dispersion and mixing on isotope concentrations are ignored. *Plummer et al.* [1993, p. 287] give an overview of current methods for age-dating groundwater, and point out, as have many others, that "because environmental tracers are dissolved solutes which are transported along with the ground water, it is usually necessary to consider effects of hydrodynamic dispersion on the modeled age." *Walker and Cook* [1991, p. 41] "show how neglecting diffusion can lead to serious underestimates of groundwater ages in unconfined aquifers where recharge rates are similarly low" when using the isotope carbon 14. *Maloszewski and Zuber* [1991] demonstrate the effects of matrix diffusion and exchange reactions on carbon 14 movement in fractured rocks and on groundwater age. *Geyh and Backhaus* [1978] examine the effects of diffusion and mixing during pumping on carbon 14 distribution and age. The diffusion of carbon 14 from a confined aquifer into adjacent aquitards and the resulting effect on interpreted ages is quantified by *Sudicky and Frind* [1981]. In their review, *Mazor and Nativ* [1992, p. 211] identify "problem areas" in the interpretation of groundwater age, including "lack of single recharge and discharge areas; ... entrapment of ground water in 'dead' volumes ... ; [and] mixing of ground water of various ages."

Incorporating the effects of transport processes other than advection allows additional information to be extracted from tracer distributions. For example, *Torgersen et al.* [1978] estimate vertical diffusivity from observed tritium/helium-3 distributions in lakes. *Weeks et al.* [1982] used fluorocarbon distribution in the unsaturated zone to estimate soil diffusion coefficients. A model of age that incorporates dispersion can be helpful in identifying the dispersive properties of the groundwater system [*Robinson and Tester*, 1984], in addition to the mean flow properties. *Egboka et al.* [1983] estimate longitudinal dispersivity from the observed tritium distribution by fitting a one-dimensional model of tritium transport. *Musgrove and Banner* [1993] use isotopic information to help quantify mixing of distinct saline waters in a regional scale flow system.

The effects of diffusion, dispersion, and mixing can be incorporated in a transport simulation of the tracer or tracers of interest, for example, tritium [*Nir*, 1964; *Simpkins and Bradbury*, 1992; *Solomon et al.*, 1993]. Tritium is an isotope of hydrogen that is incorporated in water molecules. Simulation

Paper number 95WR03401.

of tritium transport by using a model that accounts for dispersion and diffusion reflects the underlying dispersion and diffusion of water molecules, and these water molecules have different ages. Thus the age of the water itself is affected by diffusion, dispersion, and other hydrodynamic processes.

In the same way that particle tracking is used to generate groundwater age for the case of advection alone, simulation of groundwater age as affected by dispersion and mixing without resorting to separate models for separate tracers can be useful. Such an analysis complements, but does not replace, simulation of transport of the tracer of interest, which can be affected by processes, such as sorption and chemical exchange [*Fritz et al.*, 1979], that do not affect the water.

The concept of a residence-time distribution has been used for many years to describe the statistics of the lifetimes of components in a flowing reactor [*MacMullin and Weber*, 1935; *Danckwerts*, 1953]. This approach is chosen because molecules do not behave identically; rather, mixing, diffusion, and variability in flow conditions result in different paths and residence times for different molecules. Residence-time distributions have been widely applied to the analysis of chemical reaction systems [*Levenspiel*, 1972]. *Eriksson* [1958], *Bolin and Rodhe* [1973], and *Nir and Lewis* [1975] discuss the application of residence-time-distribution theory to steady and transient geophysical systems. *Robinson and Tester* [1984] use residence-time distributions determined from tracer tests to analyze dispersion in a fractured geothermal reservoir. *Campana and Simpson* [1984] apply some of these concepts to isotopic age dating of groundwater with a discrete-state (as opposed to continuous-state) compartment model in which separate areas of the groundwater system are treated as mixed reservoirs.

The underlying connection between the residence-time distribution and transport of an ideal tracer can be exploited to develop residence-time distributions corresponding to solutions of the advection-dispersion transport equation [*Danckwerts*, 1953; *Wen and Fan*, 1975; *Nauman*, 1981; *Zuber*, 1986]. *Danckwerts* [1953] describes how the residence-time distribution can be determined experimentally from the outflow concentration of a nonreactive tracer. In this context the mean age at a point, determined from temporal integrals of the concentration (described below), is affected by all of the processes accounted for in the solute-transport equation, including dispersion and mixing. The connection between resident concentration and probabilistic travel time approaches to simulating groundwater transport is described by *Shapiro and Cvetkovic* [1988] and by *Dagan and Nguyen* [1989], who discuss advantages of the travel time approach for some analyses. Recently, *Harvey and Gorelick* [1995] present a general framework for application of temporal moment-generating equations to reactive transport.

In this paper I show that the distribution of groundwater age obeys a special form of the solute-transport equation. The mean age can be simulated directly in an analytical or numerical transport model, and the result of the simulation, that is, the predicted "concentration," is the mean age. This spatial age distribution is obtained directly, without further manipulation external to the transport simulation. For steady state flow conditions the age transport equation is derived from previous results on the residence-time distribution in systems governed by the advection-dispersion equation. This form has been presented previously for analysis of mean or "local" age of air in a room during ventilation [*Sandberg*, 1981]. A more general derivation here for transient flow conditions is based on the assumption of conservation of imaginary "age mass." The method is illustrated by a numerical simulation of two regional aquifer systems, one homogeneous and the other layered.

Derivation of Mean Age Transport Equation

Residence-Time Distribution

Danckwerts [1953] defined a function $C(t)$ to characterize the residence-time distribution for molecules in a chemical reactor. This function corresponds to the concentration at the reactor exit of a solute that is injected as an impulse (unit mass in an infinitely small time period) at the reactor entrance at time zero. For one-dimensional piston flow (advection only) conditions, the function $C(t)$ is zero except at the time equal to the advective travel time of the system, at which time the function is a Dirac delta. In the case of complete mixing, $C(t)$ is an exponentially decaying function; the outflow concentration is equal to the uniform concentration within the reservoir, which decreases because of the addition of tracer-free fluid at the inflow. For transport in porous media the function C is analogous to the outflow mass flux (mass per unit time) of a column, that is, c, the concentration measured in flux [*Kreft and Zuber*, 1978]. In multidimensional systems, the mass injected on all inflow boundaries is proportional to the fluid flux across the boundary. *Levenspiel* [1972] and *Wen and Fan* [1975] present several models for transport in flowing reactors and their respective C functions.

The mean residence time in a steady-flow domain can be determined from the concentration of a tracer injected as an impulse at time zero as

$$A = \frac{\int_0^\infty tc\,dt}{\int_0^\infty c\,dt} = \frac{E}{M} \tag{1}$$

where A is the mean residence time, or the mean age of molecules, in the reactor [*Spalding*, 1958]. The numerator of (1) is a concentration weighted average time, which I will denote as E. This form is similar to the expectation of a random variable t with probability density function c. The denominator normalizes the numerator such that c divided by the time integral of c has the properties of a probability density function. This integral, which is constant and uniform for steady flow [*Spalding*, 1958], will be denoted as M. This term is uniform for multidimensional systems, provided the mass injected on inflow boundaries is proportional to fluid flux across the boundary [see *Harvey and Gorelick*, 1995].

For transport by advection and dispersion in a constant-density fluid, the concentration satisfies

$$\frac{\partial c\theta}{\partial t} = \nabla \cdot \theta\mathbf{D} \cdot \nabla c - \nabla \cdot \mathbf{q}c \tag{2}$$

where θ is the moisture content (porosity for saturated flow); $\mathbf{D}$ is the dispersion tensor; and $\mathbf{q}$ is the specific-discharge vector. A standard model of dispersion in groundwater is assumed such that the product of moisture content and the dispersion tensor, $\mathbf{D}$, is given by

$$\theta D_{ij} = (\theta D_m + \alpha_T q)\delta_{ij} + (\alpha_L - \alpha_T)\frac{q_i q_j}{q} \tag{3}$$

where D_m is the diffusion coefficient; the Kronecker delta is $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$; α_L and α_T are the longitudinal and transverse dispersivities, respectively; q_i is a component of the specific-discharge vector; and q is the magnitude of specific discharge. Multiplying (2) by time and integrating through all time gives [*Spalding*, 1958]:

$$-\theta = \nabla \cdot \theta \mathbf{D} \cdot \nabla E - \nabla \cdot \mathbf{q} E. \tag{4}$$

The left side of (4) is obtained through integration by parts of the time derivative term. Dividing by M, which is uniform and can be brought inside the spatial derivatives, and assuming porosity is also uniform, gives

$$\nabla \cdot \mathbf{D} \cdot \nabla A - \frac{1}{\theta}\nabla \cdot \mathbf{q} A + 1 = 0 \tag{5}$$

where the defined mean age $A = E/M$ has been substituted (see *Sandberg* [1981] for comparison). Thus the mean age at a point satisfies a steady state advection-dispersion transport equation with an internal source of unit (1) strength. A more general form of (5) is derived in the next section by assuming that age is conserved under mixing. This derivation also leads to natural choices for boundary conditions for (5) to complete the mathematical framework.

Computing age by particle tracking corresponds to solution of (5) without the dispersion term. In this case, particle paths are defined by the characteristics of the governing equation, and the increasing travel time corresponds to the unit source in (5). Equation (1) can be used to calculate mean age within a small volume from a particle-tracking analysis to include, for example, the mixing induced by sampling. The methods presented here for age simulation also may be useful in accounting for the effects of dispersion and mixing on other analyses involving groundwater and travel time, such as contamination from landfills [*Lee and Kitanidis*, 1993] and time-dependent capture zones [*Bair et al.*, 1990].

Conservation of Age Mass

In this section a more general form of the age transport equation is derived from mass-conservation principles. Assuming that the mean age of mixed waters is a mass-weighted average, then the mean age is analogous to a conservative solute concentration. Although age is not a directly measurable physical property, and thus this assumption cannot be verified experimentally, it seems suitable for our conceptual model that when two water masses are mixed, the mean age of the mixture is the mass-weighted average age of the mixed components. It may be possible to experimentally verify this assumption using a tracer that has an input function that varies linearly with time. A given mass of water with a mean age A can be assumed to be characterized by its "age mass," the product of the mean age and the water mass, $A\rho V$, where ρ is water density and V is water volume. Assuming that the density of the water is constant, the mean age of a two-component mixture is a volume-weighted average:

$$A = \frac{A_1 V_1 + A_2 V_2}{V_1 + V_2} \tag{6}$$

where the subscripts distinguish the components. This is completely analogous to the concentration of a conservative solute under mixing in constant-density water. This analog can be exploited to derive a governing transport equation for mean age.

A governing equation for age mass transport can be derived from a simple box balance similar to the derivation of the mass-transport equation by *Konikow and Grove* [1977]; see also *Bear* [1979]. Consider conservation of age mass in a control volume (dimensions Δx by Δy by Δz) of aquifer material. The age mass within the control volume is the product of the age (A) and the mass of water, $\theta\rho\Delta x\Delta y\Delta z$. The age mass flux (per unit area) across the boundaries of the control volume is designated $\mathbf{J}$, with components J_x, J_y, and J_z, and includes advection with the water as well as dispersive flux. Over a time step of length Δt the age mass of the water initially in the control volume increases by the product of Δt and the mass of water. Additionally, an internal net source of age mass of rate F is included to account, for example, for net exchange of age mass with separate phases. Physical processes included in F are described below. Assuming that the age mass is conserved, a difference form of a conservation equation is

$$\begin{aligned} A(t + \Delta t)\theta\rho\Delta x\Delta y\Delta z &= A(t)\theta\rho\Delta x\Delta y\Delta z + \Delta t\theta\rho\Delta x\Delta y\Delta z \\ &+ \Delta t\{\Delta y\Delta z[J_x(-\Delta x/2) - J_x(+\Delta x/2)] \\ &+ \Delta x\Delta z[J_y(-\Delta y/2) - J_y(+\Delta y/2)] \\ &+ \Delta x\Delta y[J_z(-\Delta z/2) - J_z(+\Delta z/2)]\} \\ &+ F\Delta t\Delta x\Delta y\Delta z. \end{aligned} \tag{7}$$

Dividing by the volume and the time-step length, and allowing the size of the control volume and the time step length to go to zero in the limit, gives the governing partial differential equation for mean age transport:

$$\begin{aligned} \frac{\partial A\theta\rho}{\partial t} &= \theta\rho - \frac{\partial J_x}{\partial x} - \frac{\partial J_y}{\partial y} - \frac{\partial J_z}{\partial z} + F \\ \frac{\partial A\theta\rho}{\partial t} &= \theta\rho - \nabla \cdot \mathbf{J} + F. \end{aligned} \tag{8}$$

This form of the mean age transport equation is more general than (5), in that the water density is allowed to vary, the model of dispersive flux is not specified, and the equation is transient. The mean age spatial distribution can be determined from (8) even for aquifer systems with unsteady flow, provided the flow history is known. The ability to simulate the transient evolution of groundwater age may be useful, for example, in assessing the impact of climate change on large aquifer systems.

To complete the governing equation, a description of dispersive age mass flux in terms of age, or its gradient, is needed. Here I adopt the standard model of mass flux such that $\mathbf{J}$ is composed of advection, diffusion, and dispersion modeled as Fickian diffusion. That is, $\mathbf{J}$ in (8) is replaced by

$$\mathbf{J} \approx A\rho\mathbf{q} - \theta\rho\mathbf{D} \cdot \nabla A \tag{9}$$

where $\mathbf{D}$ is the dispersion tensor and includes a diffusion term. By substitution, the governing equation becomes

$$\frac{\partial A\theta\rho}{\partial t} = \theta\rho - \nabla \cdot A\rho\mathbf{q} + \nabla \cdot \theta\rho\mathbf{D} \cdot \nabla A + F. \tag{10}$$

If the porosity and density are constant in time and uniform in space, (10) becomes

$$\frac{\partial A}{\partial t} = 1 - \nabla \cdot A \frac{\mathbf{q}}{\theta} + \nabla \cdot \mathbf{D} \cdot \nabla A + \frac{F}{\theta \rho}. \quad (11)$$

Under steady flow conditions the time derivative in (11) (or (10)) can be set to zero to derive a governing equation for steady state mean age spatial distribution. A steady state age distribution does not exist if both $\mathbf{q}$ and $\mathbf{D}$ are zero. The steady state equation derived from (11) is the same as (5), derived above from residence-time distribution theory, with an additional source term F, which is discussed below. Initial and boundary conditions for the mean age transport equation can be specified from the control-volume balance.

Boundary and Initial Conditions

The age of groundwater is relative to the time at which the water entered the system. That is, recharge to the system is assumed to have age zero. From the definition of age mass the age mass flux, the product of age and water mass flux, is also zero at all inflow boundaries. Furthermore, the age mass flux across all noflow boundaries is also zero. These conditions can be written as

$$\mathbf{J}|_{\Gamma_1} \cdot \mathbf{n} = 0 \quad (12)$$

where Γ_1 is a noflow or inflow boundary and $\mathbf{n}$ is its unit outward normal. Note that (12) uses the previous notation for total age mass flux, $\mathbf{J}$, which includes dispersion as well as advection. Thus no dispersion is assumed to occur upstream across the boundary against the direction of the incoming flow.

The boundary condition on outflow boundaries depends on the physical situation. A common assumption, which I will adopt here, is that mass flux across an outflow boundary occurs only by advection. This condition of no dispersion across the boundary can be written

$$\mathbf{D}\nabla A|_{\Gamma_2} \cdot \mathbf{n} = 0 \quad (13)$$

where Γ_2 is the outflow boundary. This condition probably is most appropriate for discharge to surface water bodies. Alternate boundary conditions for other physical situations, such as advection and dispersion into a separate aquifer outside of the simulation domain, also can be formulated, but they are not pursued here.

The general form of the governing equation for mean age transport is transient. If the flow field is steady, then the steady distribution of mean age is determined by solution of the steady state form of the governing equation, similar to (5). In this case an initial condition is not required. If the flow field is not steady, then the transient form of the governing equation for age transport must be used and an initial condition is required. That is, the initial mean age at every point within the aquifer system must be specified. For simulation times that are very long, relative to the rate of advection through the aquifer system, the groundwater age is insensitive to the initial age distribution, although the initial age is required mathematically to yield a solution.

Internal Source of Age Mass

The internal net source of age mass, F, can account for several processes. Many aquifer systems are modeled in only the horizontal dimensions because vertical head gradients and flow rates are assumed to be small. A governing equation for this case can be obtained from (5) or (11) by vertical integration across the saturated thickness. In the resulting two-dimensional model, F could include the age mass flux due to inflow from an underlying hydrogeologic unit. For example, for the case of leakage through an aquitard, F would be the product of the leakage water mass flux rate and the age of the leakage, and would be positive for the case of inflow to the aquifer of interest. Similarly, evapotranspiration from a two-dimensional horizontal model would be treated as a net sink of age mass, and F would be negative.

For the general three-dimensional model, F could represent sources and sinks of age mass due to phase or multicontinua exchange. For example, flow in fractured rock can be modeled as a two-domain system, with separate transport equations for high-permeability fractures and for the rock matrix. Exchange of water between the fractures and the rock matrix would include an exchange of age mass. For the fracture domain, F would be the product of the rate of water mass inflow from the matrix and the age of that water. The F term for the rock matrix equation would be of equal magnitude but opposite in sign.

A final example of internal age mass exchange is for unsaturated flow through partially frozen soil. In this case, some of the flowing water may freeze, acting as a sink for age mass, or stationary ice of a different age may melt, acting as a source of age mass for the flow system. Of course, as with the other examples, for the case of age mass sources such as melting ice, the age of the source water must be specified or determined from a separate, possibly coupled, mathematical model.

Low-flow or stagnant zones are not necessarily included in this internal source term but can be handled directly in the governing equation. In such zones the advective flux is small and the groundwater age is determined primarily by diffusion, which is included in the dispersion tensor $\mathbf{D}$. In the absence of diffusion, the age of water at a stagnation point in a steady flow field is by definition infinite. However, this infinite age applies only to a point which has infinitesimally small fluid volume.

Example Simulation

The mathematical theory for simulation of groundwater age developed in the previous section is applied to a regional model of groundwater flow and transport in cross section. This application demonstrates the practical simulation of groundwater age distribution using the age transport equation, both with and without dispersion. Two hypothetical aquifer systems are considered, a uniform aquifer and an aquifer system in which a high-permeability layer exists at depth. These configurations are similar to those analyzed by *Freeze and Witherspoon* [1967] in a landmark series of papers in which they used numerical flow models to study the characteristics of regional groundwater flow. The flow equation is solved using a block-centered finite-difference flow model, MODFLOW [*McDonald and Harbaugh*, 1988]. A three-dimensional method-of-characteristics solute transport model, MOC3D [*Goode and Konikow*, 1991], is modified to solve the age transport equation, including the zero-order source term for age in (10). In contrast to finite difference or finite element numerical solutions of the transport equation, MOC3D is well suited to the case of advection alone, $\mathbf{D} = 0$.

The domain geometry and boundary conditions are identical for the two hypothetical regional aquifers considered. The domain is 1 km long and 100 m thick and is discretized by 10 rows in the vertical direction, each 10 m thick, and 50 columns in the horizontal direction, each 20 m long. Noflow boundary conditions are specified on the left, right, and bottom bound-

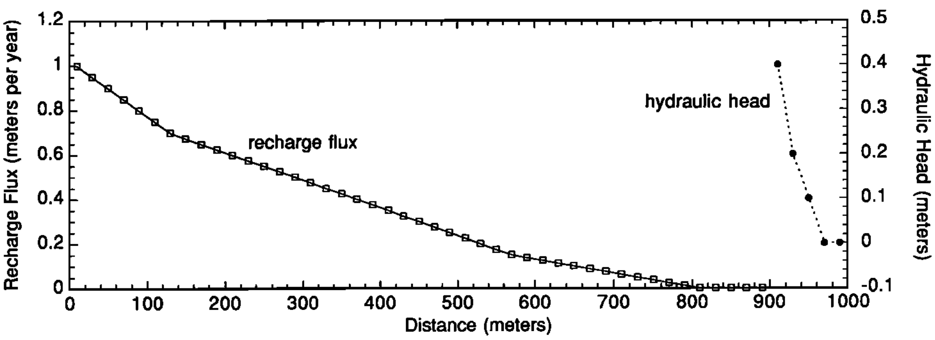

Figure 1. Flow boundary conditions on top of cross-sectional model of regional aquifer system showing specified-flux values for recharge nodes and specified-head values for discharge nodes.

aries. The top boundary represents the water table and is modeled here with a combination of specified-flux and specified-head conditions. The change in the position of the domain boundary due to movement of the water table is considered to be a minor effect and is ignored here.

The regional flow systems modeled here are similar to two considered by *Freeze and Witherspoon* [1967], but a different water table boundary condition is used. *Freeze and Witherspoon* [1967] used numerical flow models to study the effects of nonuniform hydraulic conductivity on regional groundwater flow. As previously noted by *Freeze and Cherry* [1979, p. 204], the results obtained by *Freeze and Witherspoon* [1967] were affected by the choice of specified-head boundary conditions along the entire water table, the top boundary of the flow domain. With this approach, variations in subsurface permeability lead to significant changes in recharge magnitude and distribution, without affecting water table altitudes. For the simulations here I choose boundary conditions at the opposite extreme, where the recharge is modeled as a specified-flux boundary condition (Figure 1). In reality, both water table

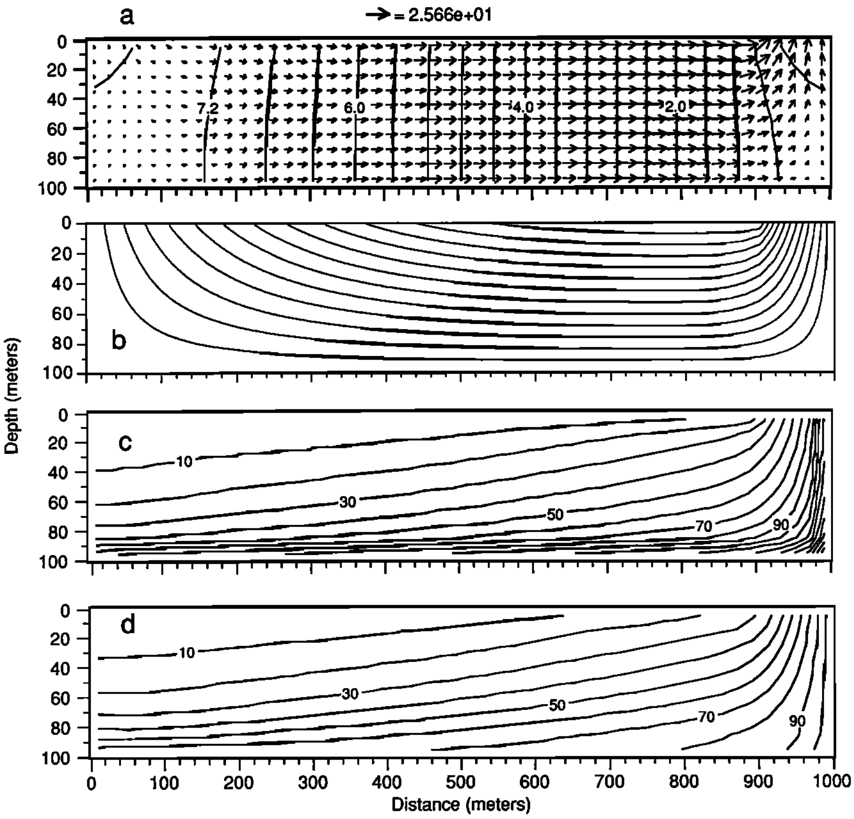

Figure 2. Results of direct simulation of groundwater age in a homogeneous regional aquifer system in which the hydraulic conductivity is 10^{-6} m/s. (a) Cross-sectional domain with hydraulic-head contours (0.4-m interval) and velocity vectors. Vertical exaggeration is 2X, and 10-row by 50-column finite difference grid used for the numerical flow and transport solutions is indicated by the small ticks on each axis. (b) Streamlines from recharge to discharge locations. (c) Contours of simulated groundwater age for advection only ($D_m = \alpha_L = \alpha_T = 0$). Contour interval is 10 years. (d) Contours of simulated groundwater age for advection and dispersion using $D_m = 1.16 \times 10^{-8}$ m²/s, $\alpha_L = 6$ m, and $\alpha_T = 0.6$ m. Contour interval is 10 years.

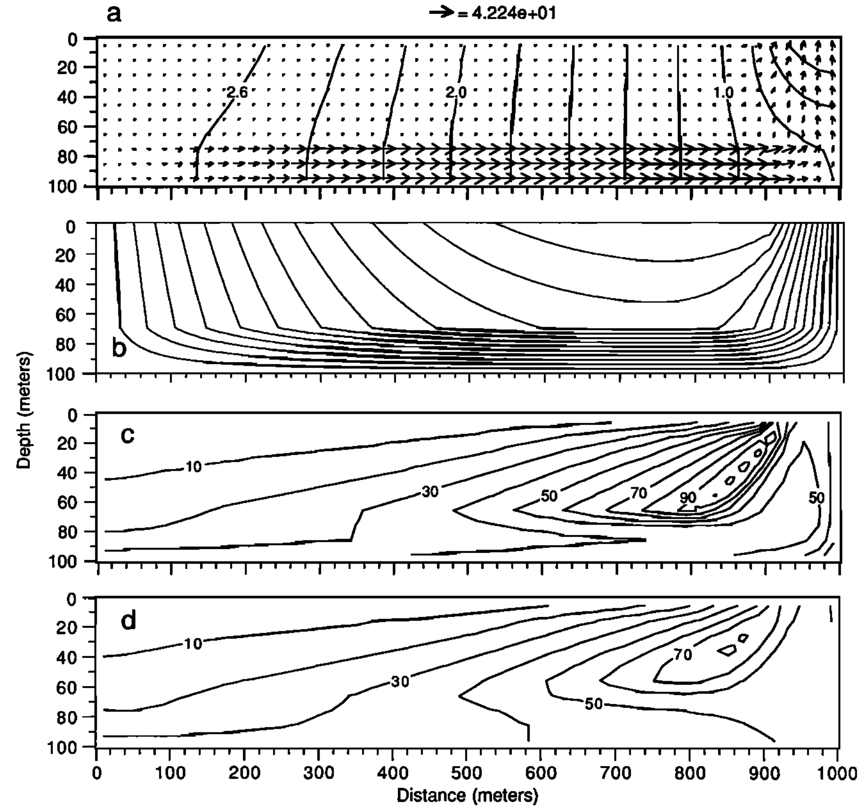

Figure 3. Results of direct simulation of groundwater age in a cross section through a layered regional aquifer system in which the bottom layer is 30 m thick with hydraulic conductivity 10^{-5} m/s, and the overlying layer is 70 m thick with hydraulic conductivity 10^{-6} m/s. (a) Cross-sectional domain with hydraulic-head contours (0.2-m interval) and velocity vectors. Vertical exaggeration is 2X, and the 10-row by 50-column finite difference grid used for the numerical flow and transport solutions is indicated by the small ticks on each axis. (b) Streamlines from recharge to discharge locations. (c) Contours of simulated groundwater age for advection only ($D_m = \alpha_L = \alpha_T = 0$). Contour interval is 10 years. (d) Contours of simulated groundwater age for advection and dispersion using $D_m = 1.16 \times 10^{-8}$ m²/s, $\alpha_L = 6$ m, and $\alpha_T = 0.6$ m. Contour interval is 10 years.

altitudes and net groundwater recharge are sensitive to subsurface permeability. Specified-head conditions similar to those of *Freeze and Witherspoon* [1967] are used at discharge locations. Thus water table altitudes are free to change in response to various hydraulic conductivity configurations, but the recharge distribution is identical in both cases. These simulations yield greater changes in hydraulic heads but smaller changes in flow rates compared to the simulations of *Freeze and Witherspoon* [1967].

Two different aquifer systems are simulated, a uniform aquifer and a layered aquifer system. The isotropic hydraulic conductivity of the homogeneous aquifer is 10^{-6} m/s and its porosity is 0.2. The hydraulic conductivity of the layered aquifer system is the same in the upper 70 m of the system, but is ten times greater, 10^{-5} m/s, in the lower 30 m. The porosity is the same for both layers, 0.2.

Figures 2a and 3a show the head distribution and groundwater flow velocities for steady state conditions, and Figures 2b and 3b show corresponding streamlines. The imposed boundary conditions and uniform properties lead to a smooth distribution of head and gradual variations in velocity for the uniform aquifer (Figure 2a). In the layered aquifer system, most flow occurs through the deep layer, where the hydraulic conductivity is greater (Figure 3a). Because the hydraulic conduc-

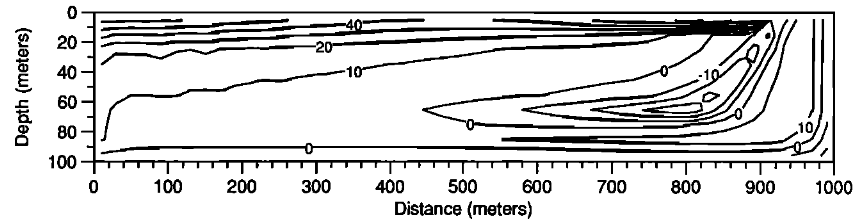

Figure 4. Percent difference in simulated age distribution between advection only and advection-dispersion ($D_m = 1.16 \times 10^{-8}$ m²/s, $\alpha_L = 6$ m, $\alpha_T = 0.6$ m) cases in a layered regional aquifer system. The percent difference in simulated age ranges from −37 to +64%.

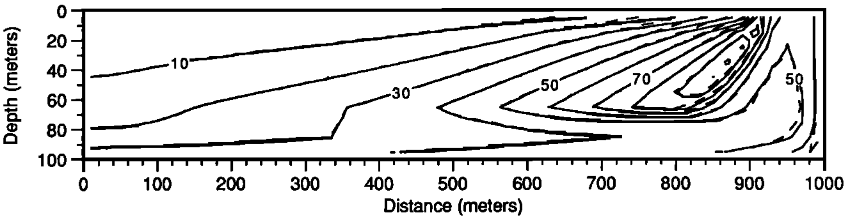

Figure 5. Contours of simulated groundwater age in a layered regional aquifer system for advection and diffusion obtained by using proposed transport equation method (solid lines), and by using a particle-tracking and random walk method (dashed lines). The diffusion coefficient is 1.16×10^{-8} m²/s, and the contour interval is 10 years.

tivity is greater, water table altitudes are lower in the layered aquifer system than in the uniform aquifer. Velocity changes abruptly at the interface, yielding refracting streamlines (Figure 3b) that contrast with the smooth streamlines throughout the uniform aquifer (Figure 2b).

The groundwater age throughout the aquifer is readily simulated by solving the advection-transport equation with the zero-order source term that accounts for aging (Figures 2c and 3c). For the homogeneous aquifer, velocities are lowest along the noflow boundaries on the left, bottom, and right. Along the right boundary, ages are the greatest at the bottom and decrease toward the discharge boundary at the top. Ages might be expected to increase continually toward the top of the right boundary, and this in fact occurs on individual streamlines. But the ages portrayed in Figures 2c and 3c are volume averages over 10- × 20-m cells. These cell-averaged values do not increase, because converging streamlines near the discharge boundary bring younger water into the cells along the right boundary. That is, the maximum age at a point is at the top right corner of the domain, but the average age for the cell containing this point is less because of the convergence of streamlines in the cell.

The age distribution in the layered aquifer system is nonuniform, and ages are greatest in the central part of the system, away from the boundaries (Figure 3c). Because the permeability of the lower layer is higher, the lowest velocities do not occur along the bottom of the system, but along the bottom of the upper layer. As with the homogeneous case, volume-averaged ages decrease toward the discharge boundary because the streamlines converge. The maximum water age in the layered aquifer system is about 90 years, whereas the maximum age in the homogeneous aquifer is about 180 years. As discussed above, the total discharge through the aquifers is identical as a result of the specified-flux boundary conditions for recharge to the water table.

This result of greater maximum ages in the homogeneous system is highly dependent on aquifer structure and boundary condition configuration. Simulation of a layered aquifer system identical to that shown here, but with the hydraulic conductivity of the lower layer decreased to 10^{-7} m/s, yields very different results. In this case the upper portion of the system is more conductive than the lower, and ages in the upper layer are similar to those in the homogeneous case. In the lower layer, however, flux and velocity are significantly reduced because of the low permeability, and very old water is present, especially toward the right side of the domain. Maximum volume-average ages for this case, with advection alone, are more than 1,000 years.

The effect of dispersion on the groundwater age distribution is easily obtained by resolving the transport equation and including dispersivities of $\alpha_L = 6$ m and $\alpha_T = 0.6$ m for the longitudinal and transverse components, respectively. In addition, a diffusion coefficient of $D_m = 1.16 \times 10^{-8}$ m²/s is added to the dispersion tensor. This diffusion coefficient is about 10 times higher than realistically expected values and is used to illustrate the maximum likely effect of diffusion in these hypothetical aquifer systems. The effect of this diffusion alone is examined below.

Dispersive and diffusive mixing of water of different ages tends to limit the maximum ages (Figures 2d and 3d). Solute dispersive flux occurs, according to the generally accepted Fick's law model, in the direction of decreasing concentration. In complete analogy the dispersive flux of age mass occurs in the direction of decreasing age, away from areas of maximum age. Where advective flux is relatively small, that is, where velocity is small, the effect of this dispersive flux on age distribution is greatest. The effect of longitudinal dispersion is mitigated in the case of age transport because along a streamline, age increases smoothly in the direction of flow due to aging. Thus the steep longitudinal gradients typically associated with an advancing solute front are not present. However, steep gradients in age are present transverse to the flow direction, particularly in the layered aquifer system (Figures 3c and 3d). In these areas of steep gradients, dispersion can have the greatest effect on groundwater age. Figure 4 is a contour map of the percent change in groundwater age, ranging from -37 to $+64\%$, owing to diffusion and dispersion for the layered case.

To examine further the relative contribution of diffusion and dispersion to the previous results and to compare the theory developed here with residence-time-distribution (particle tracking) methods, a simulation of the layered aquifer system is conducted with advection and diffusion but without dispersion. Figure 5 shows the results obtained by the theory proposed here, which are similar to those for advection alone, but exhibit a slight reduction in maximum ages. In the bottom layer, where velocities are high, diffusion has no observable effect on age distribution. Figure 5 also shows results of a particle-tracking model applied to the same problem. Particle paths and travel times are computed by linear velocity interpolation [*Goode*, 1990] and a random walk [*Kinzelbach*, 1988] to simulate diffusion. The groundwater age distribution is computed by numerical integration of (1). The close agreement between results obtained by using these alternative methods further supports the theoretical arguments presented here.

Summary

A new method is proposed to simulate groundwater age. The spatial distribution of groundwater age is governed by a transport equation that has an internal source of unit (1) strength, corresponding to the rate of aging. This governing equation is derived both from residence-time-distribution concepts and from mass-conservation principles applied to conceptual age mass. This method of groundwater age simulation falls between existing approaches of, on one hand, simulation of groundwater age as governed by advection alone, by using a particle-tracking model and, on the other hand, simulation of isotopes or chemical markers of interest in a solute transport model. Use of the theory presented here allows general simulation of groundwater age, without solute-specific modeling, by incorporating the physical effects of diffusion, dispersion, mixing, and exchange processes on age. These methods can be incorporated easily in existing models of transport in aquifers.

Acknowledgments. I am grateful to my reviewers, Michael A. Celia and Allen M. Shapiro, and to *WRR* referees Charles F. Harvey, Eileen Poeter, and D. Kip Solomon. This work was supported by the USGS Water Resources Division Graduate School Training Program and by Princeton University.

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D. J. Goode, U.S. Geological Survey, 111 Great Valley Parkway, Malvern, PA 19355.

(Received December 23, 1994; revised October 13, 1995; accepted November 1, 1995.)