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Analysing seismic-source mechanisms by linear-programming methods

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Summary. Linear-programming methods are powerful and efficient tools for objectively analysing seismic focal mechanisms and are applicable to a wide range of problems, including tsunami warning and nuclear explosion identification. The source mechanism is represented as a point in the six-dimensional space of moment-tensor components. Each observed polarity provides an inequality constraint, linear with respect to the moment tensor components, that restricts the solution to a half-space bounded by a hyper-plane passing through the origin. The intersection of these half-spaces is the convex set of all acceptable solutions. Using linear programming, a solution consistent with the polarity constraints can be obtained that maximizes or minimizes any desired linear function of the moment tensor components; the dilatation, the thrust-like nature, and the strike-slip-like nature of an event are examples of such functions. The present method can easily be extended to fit observed seismic-wave amplitudes (either signed or absolute) subject to polarity constraints, and to assess the range of mechanisms consistent with a set of measured amplitudes.

Key words: earthquake mechanism, earthquake monitoring, linear programming, seismic moment tensor

Introduction

Seismologists ordinarily determine seismic focal mechanisms from seismic-wave polarities graphically. They plot P-wave first motions on a map of the focal sphere and search manually for orthogonal nodal planes that separate compressions from dilatations. The subjectiveness of this method is a serious drawback. Possible solutions that are qualitatively different from a preferred mechanism are often overlooked. Moreover, the non-uniqueness of the nodal planes is difficult to determine and to communicate, because it involves four values (say, two dips and two strikes) connected by one constraint (orthogonality).

Several seismologists (Kasahara 1963; Jarosch 1968; Dillinger, Harding & Pope 1972; Whitcomb 1973; Khattri 1977) have tried to make this process objective, commonly by testing many possible mechanisms for consistency with the observations. They assume a double-couple mechanism, specified by the dip and strike of the fault plane and the rake angle of the slip vector, or by the plunges and trends of the P- and T-axes. Searching strategies are then necessary because the theoretical amplitudes of radiated waves are non-linear functions of these variables. Such strategies are inefficient because a large number of mechanisms must be tested. For many applications, such as automatic real-time monitoring of seismicity or tsunami warning, a fast, objective method is needed. If a moment tensor representation of the source (Gilbert 1971; Stump & Johnson 1977) is used, the amplitude functions are linear, and methods more efficient than searching can be used. G. E. Backus (unpublished) has suggested such a method, in which compressional-wave polarities are assigned artificial positive and negative amplitudes that total zero. Finding the moment tensor that best fits these amplitudes (in a least-squares sense) is then a linear inverse problem.

This report suggests another method – combining the moment tensor source representation with linear programming techniques. Linear programming differs from conventional analytical methods in that it can deal with inequalities and absolute values. It is widely used in economics because such quantities as production levels cannot be negative (Dantzig 1963), and has occasionally been applied in the physical sciences for similar reasons, for example to enforce non-negativity restrictions on density (Sabatier 1977). It has also been used to devise robust techniques for inverting erratic data, by minimizing the sum of the absolute values (L1 norm) of the data residuals rather than the sum of their squares (L2 norm) (Claerbout & Muir 1973). The application of linear programming proposed here, however, is motivated by the fact that polarity data can be expressed as linear inequalities.

Method

The moment tensor for a point source is, in general, a function of time or, equivalently, frequency:

$$\mathbf{M}(\omega) \equiv \begin{bmatrix} M_{xx}(\omega) & M_{xy}(\omega) & M_{xz}(\omega) \\ M_{xy}(\omega) & M_{yy}(\omega) & M_{yz}(\omega) \\ M_{xz}(\omega) & M_{yz}(\omega) & M_{zz}(\omega) \end{bmatrix}. \tag{1}$$

Let its six independent components be arranged as a column vector

$$\vec{\mathbf{m}}(\omega) \equiv [M_{xx}(\omega)\, M_{xy}(\omega)\, M_{yy}(\omega)\, M_{xz}(\omega)\, M_{yz}(\omega)\, M_{zz}(\omega)]^T. \tag{2}$$

The amplitude of a seismic wave or mode $\vec{\mathbf{u}}(\omega)$ is related by a linear differential operation to the moment tensor components; in the frequency domain, this operation reduces to

$$\vec{\mathbf{u}}(\omega) = \vec{\mathbf{g}}^T(\omega)\, \vec{\mathbf{m}}(\omega), \tag{3}$$

where

$$\vec{\mathbf{g}}(\omega) \equiv [g_{xx}(\omega)\, g_{xy}(\omega)\, g_{yy}(\omega)\, g_{xz}(\omega)\, g_{yz}(\omega)\, g_{zz}(\omega)]^T \tag{4}$$

is a column vector whose components are spectra of Green's functions and that depends on the type of wave and the positions of the source and receiver. Appendix A gives the components of $\vec{\mathbf{g}}$ for far-field body waves in a homogeneous medium. Then a polarity constraint takes the form (henceforth, the frequency dependence will not be explicitly indicated)

$$\vec{\mathbf{g}}^T\, \vec{\mathbf{m}} \leqslant 0 \tag{5}$$

or

$$\vec{\mathbf{g}}^T\, \vec{\mathbf{m}} \geqslant 0, \tag{6}$$

depending on its polarity. By changing the sign of the components of $\vec{g}$, the second form can be transformed into the first, and so without loss of generality we consider only constraints of the form (5).

At their simplest, linear programming methods apply to non-negative variables. Unrestricted variables, such as the moment tensor components, are treated by expressing them as the differences of non-negative variables, only one of which is allowed to differ from zero:

$$\vec{m} = \vec{m}^+ - \vec{m}^-. \tag{7}$$

A general polarity constraint now takes the form

$$\vec{g}^T \vec{m}^+ - \vec{g}^T \vec{m}^- \leqslant 0. \tag{8}$$

This expression becomes an equality constraint if we introduce a non-negative 'slack' variable s:

$$\vec{g}^T \vec{m}^+ - \vec{g}^T \vec{m}^- + s = 0. \tag{9}$$

It will also prove useful to introduce a non-negative auxiliary or 'error' variable e, so that our equation becomes, finally,

$$\vec{g}^T \vec{m}^+ - \vec{g}^T \vec{m}^- + s - e = 0. \tag{10}$$

We require that either s or e be zero. $e \neq 0$ implies $\vec{g}^T \vec{m} > 0$, whereas $s \neq 0$ implies $\vec{g}^T \vec{m} < 0$. A vector $\vec{m}$ satisfies the original constraint (5) only if $e = 0$ in (10).

Now, if there are l polarity observations, we have a system of equations of the form of (10), which can be written

$$\mathbf{A}\vec{x} = 0, \tag{11}$$

where

$$\mathbf{A} \equiv [\mathbf{G} \,|\, -\mathbf{G} \,|\, \mathbf{I} \,|\, -\mathbf{I}] \tag{12}$$

and

$$\vec{x} \equiv [\vec{m}^+ \,|\, \vec{m}^- \,|\, \vec{s} \,|\, \vec{e}\,]^T. \tag{13}$$

$\mathbf{G}$ is an $l \times 6$ matrix whose rows are the $\vec{g}$ vectors for the l observations, and $\mathbf{I}$ is the $l \times l$ identity matrix; $\vec{s}$ and $\vec{e}$ are l-vectors of the slack and error variables. Thus, if we use a moment-tensor representation of the source, the set of polarity data yields a system of linear equality constraints in non-negative variables. All acceptable solutions must satisfy this constraint system exactly. This is precisely the type of constraint that linear programming methods are designed to handle. A slight complication arises because all constants on the right sides of the inequalities are zero. To exclude moment tensors that are identically zero, a normalization restraint must be added. In the examples presented here, we have set the L1 norm of the vector $\vec{m}$ at unity. The vanishing of the right side constants also forces us to modify slightly the way the simplex algorithm handles unrestricted variables, which are expressed as the difference of two non-negative variables. If the right side constants did not vanish, then the simplex algorithm would automatically enforce the requirement that at least one of these non-negative variables must vanish. Since they do vanish, it is necessary to add an explicit test to the algorithm to ensure that this requirement is met.

Other constraints on the focal mechanism can also be incorporated, so long as they involve only linear functions of the moment tensor components. For example, the restraint $M_{xx} + M_{yy} + M_{zz} = 0$ forces the focal mechanism to be purely deviatoric (that is, to involve no volume change).

LINEAR PROGRAMMING

The problem addressed by linear programming methods is to find the n-dimensional column vector $\vec{x}$ with non-negative components that maximizes a linear objective function $v \equiv \vec{c}^T\vec{x}$ subject to the constraint $\mathbf{A}\vec{x} = \vec{b}$, where $\mathbf{A}$ is an $m \times n$ matrix, $\vec{b}$ is an m-vector, and $\vec{c}$ is an n-vector (Hadley 1962; Sakarovitch 1971). In addition, there exist quadratic programming methods, that can maximize or minimize a quadratic objective function of non-negative variables under the same types of constraints.

Linear programming problems can be solved by the simplex algorithm, which transforms a given feasible solution (one that satisfies the constraints) to increase the objective function until it reaches its maximum value. For the focal mechanism problem as formulated here, an initial feasible solution can be obtained by choosing $\vec{m}$ arbitrarily and determining $\vec{s}$ and $\vec{e}$ from the equations of the form (10) that make up the system (11). We then choose objective coefficients so as to minimize $|\vec{e}|$:

$$\vec{c} = [\vec{0}\,|\,\vec{0}\,|\,\vec{0}\,|\,-\vec{1}]^T. \tag{14}$$

This choice corresponds to finding a solution consistent with the polarity data. If no such solution exists, we obtain the solution that minimizes the L1 norm of the amplitudes for those polarities that are not satisfied. This procedure is reasonable because errors in determining polarities in the presence of noise are most likely for small signals and because errors in position on the focal sphere will cause polarity reversals only for small signals near nodes.

If a feasible solution exists, we discard the error variables (the solution will still remain feasible) and apply the simplex algorithm again with a new objective function. We discuss four possible programs for this objective function: to test observations for significance, to map out the entire solution set, to find extreme values of amplitude bias for a set of stations, and to find the limits of various physical characteristics of the solution.

TESTING OBSERVATIONS FOR SIGNIFICANCE

In general, not all the polarity observations for a given event are significant; some could be omitted without changing the set of acceptable solutions. To test a given observation for significance, we minimize the corresponding absolute amplitude. If this minimum value differs from zero, then all mechanisms consistent with the other constraints will satisfy the observation in question, and it therefore adds no new information.

MAPPING OUT THE SET OF POSSIBLE SOLUTIONS

Each polarity datum corresponds to a linear inequality in the six moment tensor components and restricts the solution to a half-space of the 6-D moment tensor space. A set of polarities thus restricts the solution to lie in the set intersection of many such half-spaces. This intersection is a convex set, and is completely defined by its vertices. We can calculate the solution corresponding to each vertex and thus give a mathematically complete description of the set of all feasible solutions. We chose all possible subsets s of five observations and for each subset we minimize

$$v \equiv \sum_{i \in s} |u_i|. \tag{15}$$

If this minimum value is zero, the solution is a vertex. As a practical matter, many possible subsets exist, and so it is more efficient to use a 'backtrack algorithm' (Wirth 1976; Page & Wilson 1979), which eliminates collections of subsets at an early stage.

In practice, there are commonly hundreds or thousands of vertices, many of which represent nearly identical solutions, and so this method is not particularly useful, despite its apparent mathematical simplicity.

FINDING EXTREME VALUES OF AMPLITUDE BIAS

The magnitude or seismic moment of an earthquake is usually determined from an average of the amplitudes of seismic waves observed at several stations. This averaging reduces the bias introduced by failure to correct for the focal mechanism, but it does not completely eliminate such bias; observations near nodes or maxima of the radiation pattern may still distort the average. To evaluate the magnitude of this bias, we can seek solutions which maximize or minimize the average value by using the objective function defined in equation (15), with s taken to be the set of seismic waves to be averaged. Note that it is not necessary that the waves averaged be among those which supplied the polarity data.

An important special case is the problem of tsunami warning. If the tsunami generation process is linear (as submarine landsliding, for example, is not) then, using the objective function just described, we can, for a given scalar seismic moment, determine the largest (or smallest) possible tsunami amplitudes consistent with a set of polarity observations. Furthermore, by using seismic-wave amplitude observations, (as explained below under 'further extensions') we can take account of the effects of both seismic moment and focal mechanism, and determine the extreme tsunami amplitudes consistent with a data set containing amplitude and (optionally) polarity observations.

FINDING PHYSICALLY EXTREME SOLUTIONS

Any physical parameter of a seismic event that can be expressed as a linear function of the moment tensor components can be used as an objective function. For example, the dilatation (volume change) of a source is the trace of the moment tensor,

$$\Delta = M_{xx} + M_{yy} + M_{zz}, \tag{16}$$

and may be used as an objective function to determine the most 'explosive' or 'implosive' consistent mechanism. Similarly, the thrust-like nature of an event can be defined in terms of the amplitude of the compressional wave radiated downward, which is proportional to M_{zz} (see Appendix A), the horizontal extension in the x-direction can be defined as M_{xx}, etc. Furthermore, it is possible to define objective functions that depend linearly on either the algebraic (signed) or absolute values of the moment tensor components. Appendix B gives several examples of useful physically motivated objective functions.

Example

We illustrate the above method by using long-period first-motion data for the Imperial Valley, California earthquake of 1979 October 15, obtained from 12 stations in the Global Digital Seismograph Network (GDSN) (Peterson *et al.* 1976). First motions can be determined for P-waves at six stations, and for horizontally polarized shear waves at all 12 (Julian, Zirbes & Needham 1982). Table 1 lists the first motions, and Fig. 1 shows their positions on the focal sphere.

Table 1. Long-period first motions of the Imperial Valley earthquake of 1979 October 15.

Station code	Epicentral distance (°)	Ep > Sta azimuth (°)	Takeoff angle (°)	Phase	First motion
ANMO	7.7	70	45.8	*P*	+ *
			45.8	*SH*	–
BOCO	47.6	117	24.0	*P*	+ *
			24.0	*SH*	+ *
ZOBO	66.4	130	19.5	*P*	+ *
			19.5	*SH*	+ *
KONO	77.5	25	17.0	*P*	– *
			17.0	*SH*	– *
MAJO	82.8	309	15.8	*P*	+ *
			15.8	*SH*	+
GUMO	90.8	287	14.1	*P*	+
			14.1	*SH*	+ *
SNZO	97.8	226	14.0	*SH*	– *
TATO	101.2	309	14.0	*SH*dif	+
CTAO	107.4	257	14.0	*SH*dif	– *
CHTO	119.2	322	14.0	*SH*dif	+
BCAO	122.9	59	14.0	*SH*dif	–
NWAO	136.2	255	14.0	*SH*dif	–

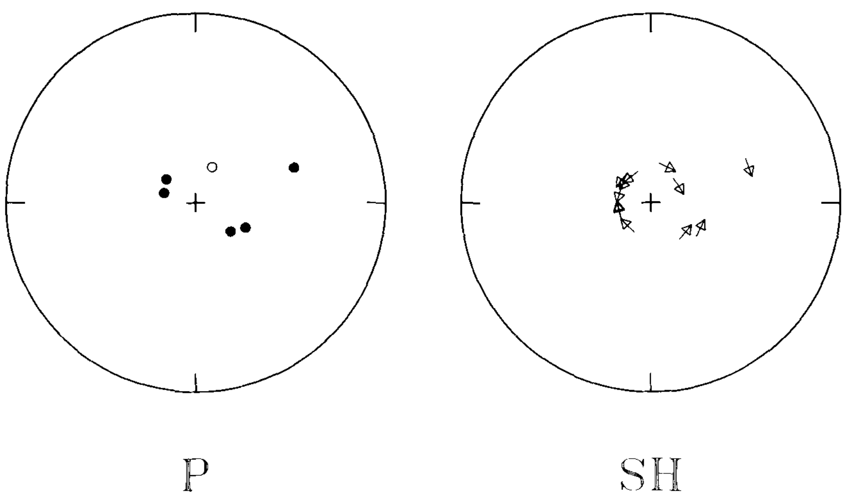

Figure 1. Equal-area projection of lower focal hemisphere, showing first-motion data from Table 1. *P*-wave polarities are indicated by pluses (compressions) and circles (dilatations), and *SH*-wave polarities by arrows.

The observations are mapped on to the focal sphere using a preliminary location supplied by the US Geological Survey, National Earthquake Information Service (lat. 32.641°N, long. 115.325°W; depth, 10 km) and the continental earth model PEMC (Dziewonski, Hales & Lapwood 1975). The compressional- and shear-wave speeds at the focus in this model are 5.8 and 3.45 km s^{-1}.

Eleven of these first-motion data are significant (asterisks, Table 1). Table 2 gives the principal axis representations of the moment tensors for three extreme focal mechanisms,

Table 2. Extreme solutions

	Axis	Moment	Plunge (°)	Trend (°)
Most thrust-like	*T*	0.366	23	268
	I	0.052	65	108
	P	−0.418	8	1
Most normal	*T*	0.450	2	291
	I	−0.036	87	161
	P	−0.414	3	21
Least dip-slip	*T*	0.450	2	292
	I	0.000	86	163
	P	−0.451	3	22

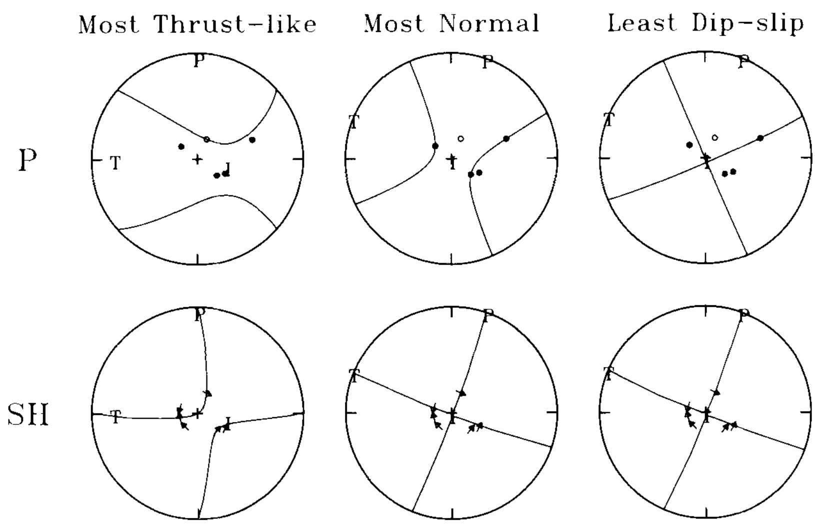

Figure 2. Comparison of predicted and observed first motions for three extreme solutions listed in Table 2. Only significant data (asterisks, Table 1) are shown, using the same symbols as in Fig. 1. Theoretical nodal curves are shown for *P*-waves (solid) and *SH*-waves (dashed). P, T, and I mark the positions of the principal axes of the moment tensor.

and Fig. 2 compares the predicted polarities with those observed. Because no amplitude information is used in this analysis, the principal moments in Table 2 are determined only within an arbitrary factor.

These three mechanisms have extreme values for the amplitude of the compressional wave radiated downward. The mechanism labelled 'most thrust-like' has the largest (i.e. most compressional) amplitude, the 'most normal' mechanism has the smallest (most rarefactional) amplitude, and the 'least dip-slip' mechanism has the smallest absolute amplitude (zero, in this case).

Because the mechanisms are not forced to be double couples, the nodal curves are no longer orthogonal great circles on the focal sphere (although the 'least dip-slip' mechanism is very close to this case). All the focal mechanisms are similar, and the smallness of their intermediate principal moments shows that they are predominantly double couples.

The use of shear-wave polarities has not been common in focal-mechanism studies and so deserves further comment. When data from properly oriented seismometers are available, the determination of shear-wave polarities is feasible. (With digital data, synthetic traces with any orientation can be generated numerically.) First motions for vertically polarized waves are hard to determine, however, because the signal is commonly contaminated by compressional-wave energy generated by mode conversion near the receiver; this problem also complicates attempts to measure polarization angles for shear waves. For horizontally polarized waves, however, no mode conversion occurs, and first motions are generally clearer and have better signal-to-noise ratios than those for compressional waves. In principle, surface-wave polarities could also be used, although precise knowledge of the regional phase velocities would be required, and the excitation coefficients depend strongly on focal depth.

Comparison with searching methods

In order to compare the efficiency of linear programming and searching methods, a number of benchmark tests were performed using the searching program FOCAL1, of Whitcomb and Garmany (Whitcomb 1973) and a program that uses the linear programming method described in this paper. Both programs are written in FORTRAN, and were compiled with the portable FORTRAN 77 compiler (Feldman & Weinberger 1983) and run on a Digital Equipment Corporation PDP-11/70 minicomputer under the Unix operating system.

It should be emphasized that the results of a benchmark test such as this depend not only on the algorithms used, but also on details of how the computer programs are written. Nevertheless, the results in this case are probably not misleading. The searching program is optimized in many ways, whereas the linear-programming program uses a very simple version of the simplex algorithm, which is far from optimal. For example, it takes no advantage of the fact that the matrices involved are quite sparse. Therefore, this benchmark test is, if anything, probably biased in favour of the searching program. Table 3 gives execution times of both programs for tests with various numbers of polarity observations. The linear programming times are divided into two parts: the time to determine which data are significant, and the time to find 10 extremal solutions consistent with the significant data. (Fewer than 10 extremal solutions are adequate to determine the range of possible solutions for any geophysical purpose.) The searching program provides for coarse and medium-resolution searching, as well as normal resolution; times for all three options are shown in the table.

The effort required for the linear programming method to identify significant data obviously increases rapidly with the number of observations. This is a defect of the organization of the current version of the computer program, and is easily correctable. At present, all the observations are used to generate the constraint matrix and then each observation is tested for significance, which usually requires several pivot operations on the matrix per observation. Since, typically, most of the observations are not significant, the matrix is several times larger than necessary, and the pivot operations are unnecessarily slow. If observations were tested first, and entered only if they were significant, the amount of

Table 3. Benchmark test results.

No. of data	Execution time (s)			
	Linear programming	Searching		
		Coarse	Medium	Fine
15	2.1 + 6.8 = 8.9	30.1	35.2	157.8
30	6.1 + 6.4 = 12.5	31.4	48.4	229.5
60	43.1 + 10.3 = 53.4	39.0	68.4	367.5

labour involved would be greatly reduced and would not increase rapidly with the number of data. (It would, however, still be necessary to retest the significance of each retained observation using the final constraint matrix, in case data that were entered after it had rendered it redundant. Data identified as redundant during construction of the constraint matrix are guaranteed to be truly redundant, but the process cannot necessarily identify all redundant observations until the matrix is complete.)

Extensions of the method

So far, we have dealt only with polarity data, which we have assumed to be perfect and free from errors, and we have considered completely general moment tensor sources and sources that were constrained to be purely deviatoric. There are several ways in which the linear programming method can be extended to overcome these limitations. We will briefly discuss three of these extensions here. The actual implementation and testing of them is left for the future.

DOUBLE-COUPLE CONSTRAINT

As we have seen, requiring an earthquake mechanism to be purely deviatoric is equivalent to constraining the trace of the moment tensor to vanish, a constrain that is linear and easily imposed. To restrict solutions to be double couples, it is also necessary to constrain the determinant of the moment tensor to vanish. This constraint is cubic and must be dealt with by linearization and iteration. Such an extension, while undoubtedly possible, would probably involve sacrificing many of the advantages of a linear formulation. For example, convergence of the iterative process would not be assured, and solutions might not be unique and could depend in an unpredictable way upon the initial solution chosen.

USING AMPLITUDES

So far, we have considered only polarity data. However, L1-norm fitting of observed amplitudes amounts to minimizing a linear objective function, and so achievement of such a fit consistent with a set of observed polarities is a linear-programming problem and can be incorporated into the present method. The robustness of L1-norm fitting makes this method particularly attractive for use with seismic-wave amplitude observations, which commonly exhibit much scatter. Moreover, it is possible to deal with both signed-amplitude observations, for which the fitting error is $|u_{\text{obs}} - u_{\text{calc}}|$, and absolute-amplitude observations, for which it is $\big|u_{\text{obs}} - |u_{\text{calc}}|\big|$. Least-squares fitting methods, in contrast, can deal with amplitudes only if the polarities are known, which is frequently not the case, for example with noisy or dispersed signals.

To fit a signed amplitude, we add to our system of constraint equations, (11), an equation much like (10):

$$\vec{\mathbf{g}}^T \vec{\mathbf{m}}^+ - \vec{\mathbf{g}}^T \vec{\mathbf{m}}^- + s - e = u_{\text{obs}} \tag{19}$$

then, since

$$u_{\text{calc}} \equiv \vec{\mathbf{g}}^T \vec{\mathbf{m}}^+ - \vec{\mathbf{g}}^T \vec{\mathbf{m}}^- \tag{20}$$

and since s and e are non-negative and one of them is zero, we have

$$s + e = |u_{\text{obs}} - u_{\text{calc}}|. \tag{21}$$

Thus to fit a set of signed amplitudes, we take as the objective function the sum of the s and e terms for the observations.

To fit an absolute-amplitude observation, we append two equations with two new variables, x and y, to the system (11):

$$\vec{g}^T\vec{m}^+ - \vec{g}^T\vec{m}^- + s - e = 0 \tag{22}$$

$$s + e + x - y = u_{\text{obs}}. \tag{23}$$

x and y are non-negative and we require one of them to be zero. By the same reasoning used above, we obtain from (22)

$$s + e = |u_{\text{calc}}|, \tag{24}$$

and from (23)

$$x - y = u_{\text{obs}} - |u_{\text{calc}}| \tag{25}$$

and

$$x + y = \big|u_{\text{obs}} - |u_{\text{calc}}|\big|. \tag{26}$$

In this case, the correct objective function is the sum of the x and y terms for the observations.

Since observed data always are contaminated by errors, it is just as important to determine how tightly a focal mechanism is constrained by a data set as it is to find the best-fitting mechanism. To assess the uniqueness of the mechanism, we can place bounds on the allowable misfit for each amplitude observation,

$$u_{\min} \leqslant u_{\text{calc}} \leqslant u_{\max}, \tag{27}$$

and investigate the range of solutions that are consistent with a set of such constraints. The bounds (27) lead to equations similar to (19):

$$\vec{g}^T\vec{m}^+ - \vec{g}^T\vec{m}^- + r = u_{\max} \tag{28}$$

$$\vec{g}^T\vec{m}^+ - \vec{g}^T\vec{m}^- - s = u_{\min}, \tag{29}$$

where r and s are new non-negative slack variables. (Note that the polarity constraint of equation (9) may be thought of as a special case of an amplitude constraint with either $u_{\max}$ or $u_{\min}$ equal to zero.) Similarly, an unsigned amplitude constraint

$$u_{\min} \leqslant |u_{\text{calc}}| \leqslant u_{\max}, \tag{30}$$

is equivalent to the equations

$$\vec{g}^T\vec{m}^+ - \vec{g}^T\vec{m}^- + s - e = 0 \tag{31}$$

$$s + e + f = u_{\max} \tag{32}$$

$$s + e - g = u_{\min}, \tag{33}$$

where s, e, f, and g are, as always, non-negative slack and error variables.

Thus there are two ways of dealing with amplitudes by linear programming methods, and they can be applied to any combination of signed- and absolute-amplitude observations and optional polarity constraints. We may determine the mechanism that fits the amplitudes best (in terms of the L1-norm), or we may place bounds on the allowable (signed or absolute) theoretical amplitudes for the observations (or on their sum) and then determine the set of mechanisms consistent with these constraints.

An important application of these techniques for analysing amplitude observations arises in the problem of determining the seismic moment of an explosion. In addition to the explosion itself, the moment tensor usually contains a deviatoric component caused by the triggered release of tectonic strain, which tends to bias estimates of the moment of the explosion. The linear programming method can deal with this problem by computing the largest and smallest possible values of the explosive component of the moment tensor for all mechanisms that satisfy the amplitude observations within the observational errors.

IMPERFECT POLARITY DATA

Another assumption we have made so far is that all the polarity data are correct; solutions have been considered acceptable only if they are consistent with all the polarities. It is simple, however, to relax this restriction: instead of insisting that all the error variables e in the system (11) must vanish, we ask only that their sum be less than some limit E. After an initial solution satisfying this condition is obtained by using the simplex algorithm with the objective function (14), we add an equation to ensure that it remains satisfied:

$$e_1 + e_2 + \ldots + e_l + s_{l+1} = E \tag{34}$$

where s_{l+1} is a new non-negative slack variable.

Conclusions

Applied to the moment tensor representation of a seismic source, linear programming is a powerful and efficient method for analyzing seismic-wave polarity and amplitude observations. It can quantitatively and objectively delimit the set of focal mechanisms consistent with a set of polarity observations with a fraction of the effort required by searching methods. It can also find focal mechanisms consistent with observed polarities that best fit (in terms of the L1 norm) a set of amplitude observations, even when some (or all) of these amplitudes are of unknown sign. The speed and objectivity of the linear-programming approach ideally suits it to automatic rapid determination of earthquake mechanisms, for example as part of a real-time tsunami warning or earthquake prediction system.

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Appendix A: excitation of body waves by point sources

The amplitude of any type of seismic wave or mode excited by a point source is a linear function of the moment-tensor components (see equation (3)). Table A1 lists the Green's functions for far-field body waves in a homogeneous isotropic medium as functions of the departure angle i (zero denotes straight downward) and departure azimuth ζ.

Table A1. Body-wave Green's functions.

	P	SV	SV
g_{xx}	$\sin^2 i \cos^2 \zeta$	$-\frac{1}{2}\sin 2i \cos^2 \zeta$	$\frac{1}{2}\sin i \sin 2\zeta$
g_{xy}	$\sin^2 i \sin 2\zeta$	$-\frac{1}{2}\sin 2i \sin 2\zeta$	$-\sin i \cos 2\zeta$
g_{yy}	$\sin^2 i \sin^2 \zeta$	$-\frac{1}{2}\sin 2i \sin^2 \zeta$	$-\frac{1}{2}\sin i \sin 2\zeta$
g_{xz}	$\sin 2i \cos \zeta$	$-\cos 2i \cos \zeta$	$\cos i \sin \zeta$
g_{yz}	$\sin 2i \sin \zeta$	$-\cos 2i \sin \zeta$	$-\cos i \cos \zeta$
g_{zz}	$\cos^2 i$	$\frac{1}{2}\sin 2i$	0

A common factor of $\delta(t - R/v)/(4\pi\rho v^3 R)$ has been omitted, where ρ is the density at the source, v is the seismic wave (P or S, as appropriate) speed at the source, and R is the distance to the observation point; δ is the Dirac delta function. The moment-tensor components are expressed in a right-handed Cartesian coordinate system, with the z-axis directed downward. i is the angle between the ray direction and the $+z$-axis, and ζ is measured from the $+x$-axis toward the $+y$-axis. If the $+x$-axis is directed northward, then these definitions agree with the conventional definitions of take-off angle and azimuth. The compressional (P)-wave amplitude is taken as positive for particle motion away from the source. Shear-wave amplitudes are positive for particle motion in the direction of decreasing i for vertically polarized (SV) waves, and in the direction of decreasing ζ for horizontally polarized (SH) waves.

Appendix B: some physically motivated objective functions for moment tensors

Any linear function of the variables in a linear programming problem may be used as an objective function. In the case of focal mechanisms, the variables consist of the moment tensor components and the slack and error variables (equation (13)). Furthermore, each moment tensor component is expressed as the difference of two non-negative numbers, one of which is zero, so it is possible to design objective functions that depend on absolute values of moment tensor components.

An important class of objective functions is those that depend only on the moment tensor components, and seek to maximize or minimize some physical characteristic of the focal mechanism. Table B1 gives the coefficients of a few such functions that have been found to be useful.

Table B1. Objective coefficients.

	M_{xx}	M_{xy}	M_{yy}	M_{xz}	M_{yz}	M_{zz}
Explosive	1	0	1	0	0	1
	-1	0	-1	0	0	-1
Thrust-like	0	0	0	0	0	1
	0	0	0	0	0	-1
Horizontal	$\cos^2 \zeta$	$\sin \zeta \cos \zeta$	$\sin^2 \zeta$	0	0	0
Extension	$-\cos^2 \zeta$	$-\sin \zeta \cos \zeta$	$-\sin^2 \zeta$	0	0	0
Vertical	-1	0	-1	0	0	2
CLVD	1	0	1	0	0	-2
Horizontal	$3 \cos^2 \zeta - 1$	$3 \sin \zeta \cos \zeta$	$3 \sin^2 \zeta - 1$	0	0	-1
CLVD	$-3 \cos^2 \zeta + 1$	$-3 \sin \zeta \cos \zeta$	$-3 \sin^2 \zeta + 1$	0	0	1
Vertical	0	0	0	-1	-1	-1
strike-slip	0	0	0	-1	-1	-1

For each function, there are two rows of six coefficients each; coefficients in the upper row apply to the vector $\vec{m}^+$ and the lower row applies to $\vec{m}^-$ (see equations (7) and (13)). In other words, the coefficient in the upper row applies when the moment tensor component is positive, and the coefficient in the lower row applies (to the *absolute value*) when the component is negative. The variable ζ gives the azimuth of the direction of horizontal extension in the two cases where it appears.

Most of these cases attempt to force the moment tensor (expressed as a six-vector) into some particular direction by maximizing its dot product with a specified vector. In these cases, the corresponding coefficients in the two rows are equal in magnitude and opposite in sign, so that the algebraic value of the component is dealt with. Examples are the 'explosive' case, which maximizes the trace of the moment tensor, $M_{xx} + M_{yy} + M_{zz}$, and the 'thrust-like' case, which maximizes M_{zz}. M_{zz} is proportional to the amplitude of the downward-radiated compressional wave (see Appendix A), or equivalently to the vertical extension accompanying the event. Similarly, the 'horizontal extension' case maximizes the extension in any specified horizontal direction. The 'CLVD' cases seek solutions that are like 'Compensated Linear Vector Dipoles' (Knopoff & Randall 1970) with particular orientations. For each of these cases, it is possible to reverse the sign of all the coefficients, so as to minimize algebraically, rather than maximize, the relevant quantity.

The 'vertical strike-slip' case, on the other hand, deals with *absolute values* of the moment tensor components, seeking to minimize $|M_{xz}| + |M_{yz}| + |M_{zz}|$ because, for a vertical strike-slip faulting mechanism, all these components vanish. In this case, the coefficients in the two rows are identical.

Obviously, this table is incomplete; the number of potentially useful objective functions is virtually limitless.